

# Cargas Combinadas - PILARES

# FLEXÃO PARTE 4

①

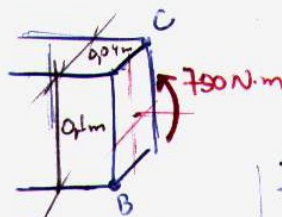
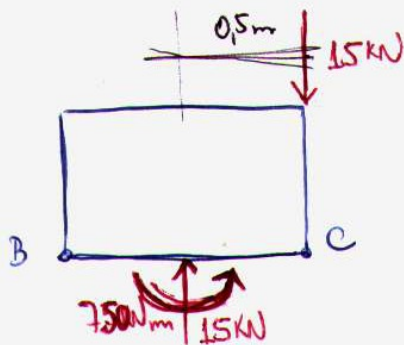
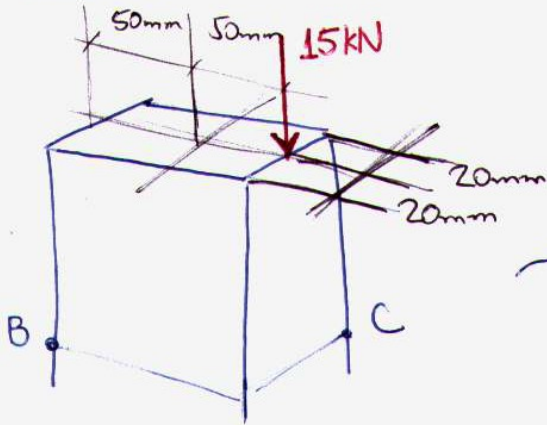
5<sup>o</sup> ed. 325-340 Hibbeler

→ Princípio da Superposição

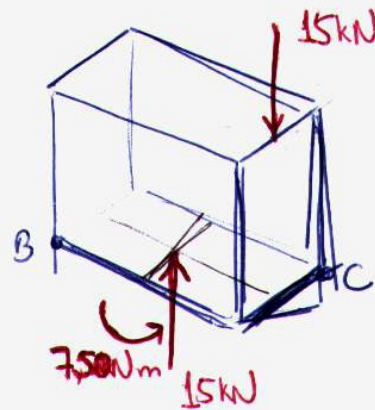
Saint-Venant :   
 • relação linear tensão x deformação   
 • geometria do elemento não pode variar significativamente

→ revisão   
 { normal (axial)   
 momento fletor

a)

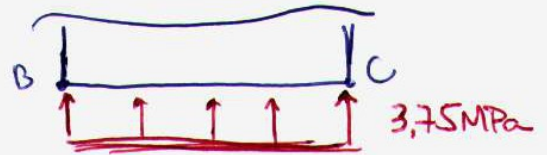


## I) CARGAS INTERNAS



## II) FORÇA NORMAL

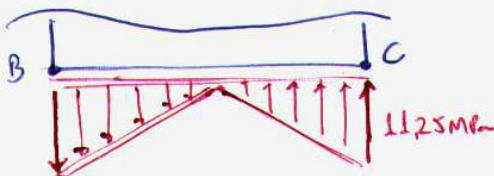
$$\sigma = \frac{P}{A} = \frac{15000}{0,1 \times 0,04} = 3,75 \text{ MPa}$$



## III) MOMENTO FLETOR

$$\sigma_{\text{máx}} = \frac{M \cdot y}{I}; I = \frac{b \cdot h^3}{12}; y = \frac{h}{2}$$

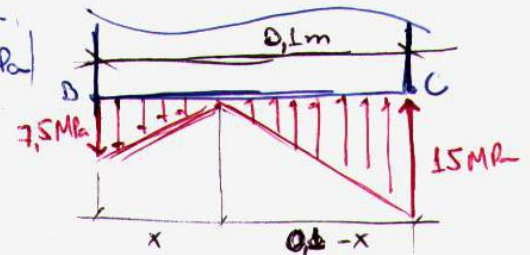
$$\sigma_{\text{máx}} = \frac{M \cdot \frac{h}{2}}{\frac{b \cdot h^3}{12}} = \frac{M \cdot 6}{b \cdot h^2} = \frac{750 \cdot 6}{0,04 \cdot (0,1)^2} = 11,25 \text{ MPa}$$



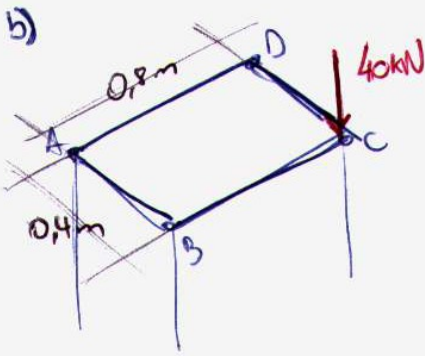
## IV) SUPERPOSIÇÃO

$$EM_B: \sigma = -3,75 + 11,25 = 7,5 \text{ MPa}$$

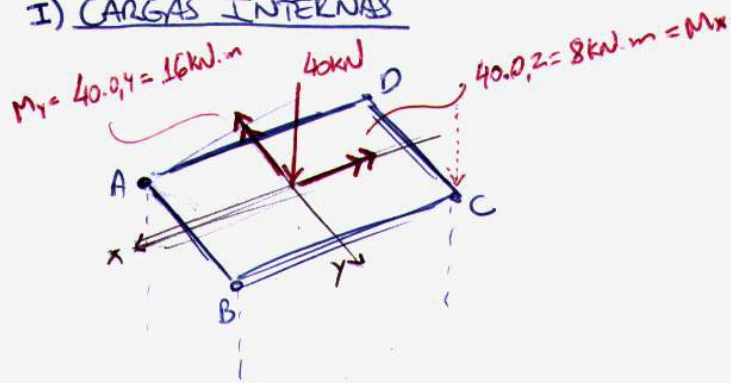
$$EM_C: \sigma = -3,75 - 11,25 = -15 \text{ MPa}$$



$$\frac{7,5}{x} = \frac{15}{(0,1-x)} \Rightarrow 15x = 0,75 - 7,5x \Rightarrow 22,5x = 0,75 \Rightarrow x = 3,33 \times 10^{-2} \text{ m}$$



I) CARGAS INTERNAS



II) FORÇA NORMAL

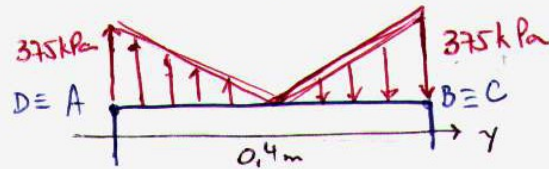
$$\sigma = \frac{P}{A} = \frac{40.000}{0,8 \cdot 0,4} = 125 \text{ kPa}$$



III) MOMENTO Mx

$$\sigma = \frac{M_y}{I}; I = \frac{b \cdot h^3}{12}; \gamma = \frac{h}{2}$$

$$\sigma = \frac{6M}{b \cdot h^2} = \frac{6 \cdot 8.000}{0,8 \cdot (0,4)^2} = 375 \text{ kPa}$$



IV) MOMENTO FLETOR My

$$\sigma = \frac{M \cdot y}{I} \Rightarrow \sigma = \frac{6 \cdot M}{b \cdot h^2} = \frac{6 \cdot 16.000}{0,4 \cdot (0,8)^2} = 375 \text{ kPa}$$



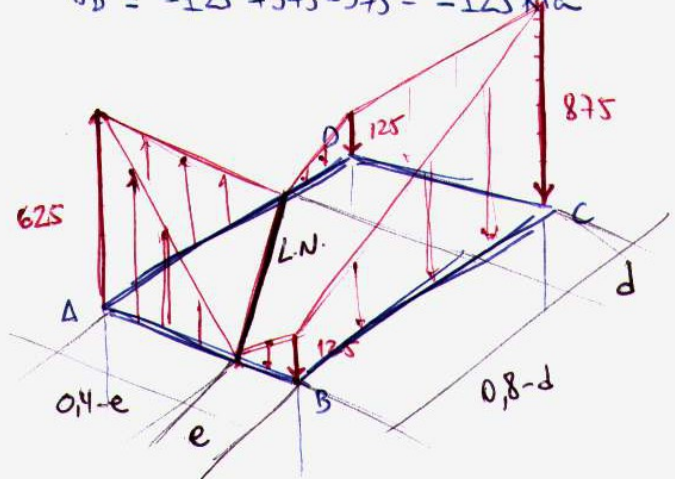
V) SUPERPOSIÇÃO

$$\sigma_A = -125 + 375 + 375 = +625 \text{ kPa}$$

$$\sigma_B = -125 - 375 + 375 = -125 \text{ kPa}$$

$$\sigma_C = -125 - 375 - 375 = -875 \text{ kPa}$$

$$\sigma_D = -125 + 375 - 375 = -125 \text{ kPa}$$



VI) EXCENTRICIDADES

$$\frac{125}{d} = \frac{625}{(0,8-d)} \Rightarrow 625d = 100 - 125d$$

$$750d = 100 \Rightarrow d = 0,13 \text{ m}$$

$$\frac{125}{e} = \frac{625}{(0,4-e)} \Rightarrow 625e = 50 - 125e$$

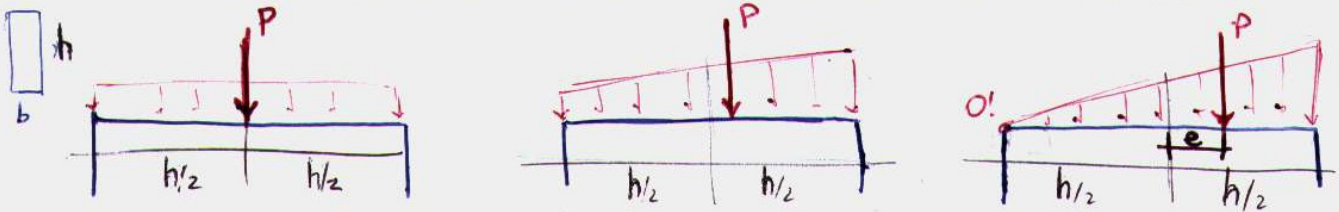
$$750e = 50 \Rightarrow e = 0,067 \text{ m}$$

# NÚCLEO CENTRAL

(3)

QUAL A MAIOR EXCENTRICIDADE PARA A QUAL N APARECE TRAÇÃO?

→ IMPORTANTE PARA PEDRA OU CONCRETO NO ARMADO EM ARCOS E PILARES



$$\sigma_{\min} = -\frac{P}{A} + \frac{M \cdot c}{I_x}; \quad I_x = \frac{b \cdot h^3}{12} = \frac{A \cdot h^2}{12}$$

$$M = P \cdot e$$

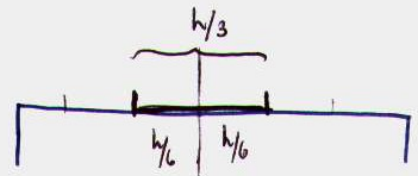
$$c = \frac{h}{2}$$

$$\sigma_{\min} = -\frac{P}{A} + \frac{P \cdot e \cdot \frac{h}{2} \cdot \frac{12}{A h^2}}{1} = -\frac{P}{A} + \frac{P}{A} \cdot \frac{6e}{h} \leq 0 \leftarrow \text{DESEJAMOS ISSO}$$

Logo

$$\sigma_{\min} \leq 0 \text{ se } \frac{P}{A} \cdot \frac{6e}{h} \leq \frac{P}{A} \Rightarrow \sigma_{\min} \leq 0 \text{ se } \frac{6e}{h} \leq 1$$

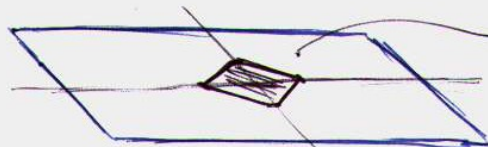
$$\text{ou } \sigma_{\min} \leq 0 \text{ se } \boxed{e \leq \frac{h}{6}}$$



ESSE CONCEITO PODE SER ESTENDIDO PARA AMBAS AS DIREÇÕES

$$\sigma_{\min} = -\frac{P}{A} + \frac{P}{A} \cdot \frac{6e_x}{h} + \frac{P}{A} \cdot \frac{6e_y}{b} \leq 0$$

$$\Rightarrow \frac{P}{A} \cdot \frac{6e_x}{h} + \frac{P}{A} \cdot \frac{6e_y}{b} \leq \frac{P}{A} \Rightarrow \frac{6e_x}{h} + \frac{6e_y}{b} \leq 1$$



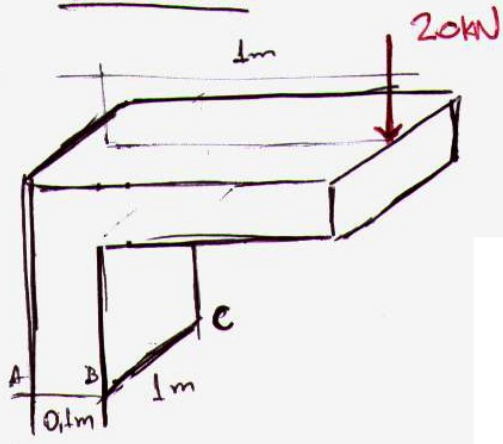
NÚCLEO CENTRAL

EXERCÍCIOS: 8.15, 8.21, 8.22

EXTRAS: 8.25, 8.31, 8.62

EXERCÍCIO

4



QUAL A DISTRIBUIÇÃO DE  
TENSOES NORMAIS EM ABCD