



PESQUISA OPERACIONAL II

O PROBLEMA DO FLUXO MÁXIMO: MÉTODO DE FORD-FULKERSON

Prof. Dr. Daniel Caetano

2019 - 1

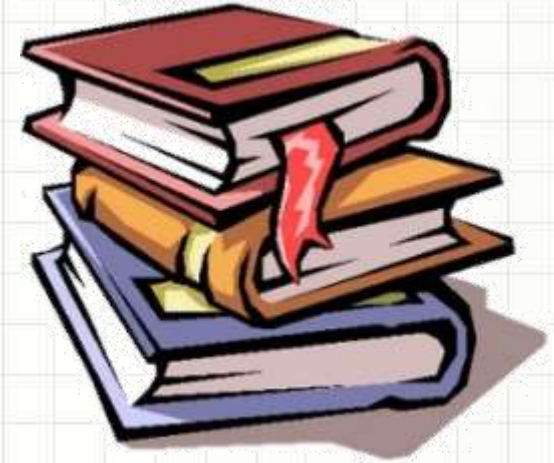
Objetivos

- Compreender o Problema do Fluxo Máximo
- Compreender o método de Ford-Fulkerson
- Compreender o tratamento das incertezas

- **Atividade Aula 10 – SAVA!**



Material de Estudo



Material

Acesso ao Material

Apresentação

<http://www.caetano.eng.br/>
(Pesquisa Operacional II – Aula 10)

Biblioteca Virtual

Pesquisa Operacional (Taha) – Seção 6.4

Recursos na Web

Algoritmo de Ford-Fulkerson:
<https://tinyurl.com/y6hjhl5s>



O PROBLEMA DO FLUXO MÁXIMO

Problema do Fluxo Máximo

- escoar a produção por um conjunto de vias
 - Quanto é o máximo que consigo levar de A a B?
 - Comum em linhas de trem
- Múltiplos caminhos, capacidades diferentes
- Quanto é o fluxo máximo de A a B?
 - Água
 - Energia
 - Gás

Modelagem Matemática

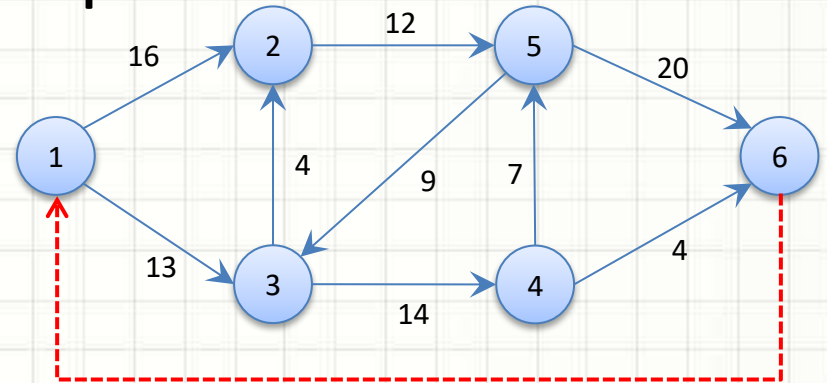
- Maximizar fluxo entre os pontos 1 e 6
- Modelo Completo
- F.O.:

$$[max] \sum_j x_{oj} - \sum_i x_{io}$$

- S.A.:

$$\sum_{ij \in E} x_{ij} - \sum_{jk \in E} x_{jk} = 0$$

$$x_{ij} \leq c_{ij}$$

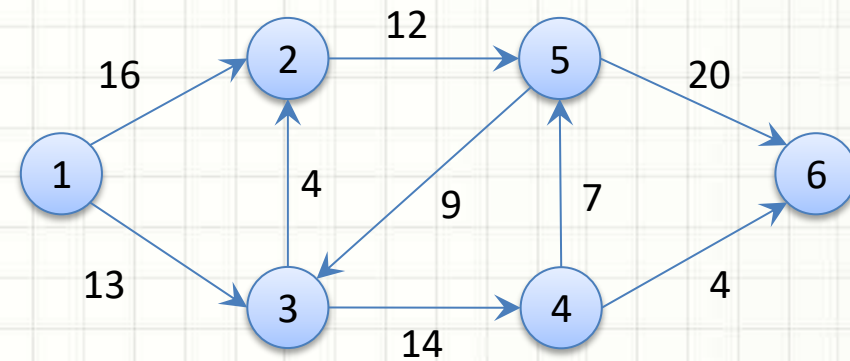
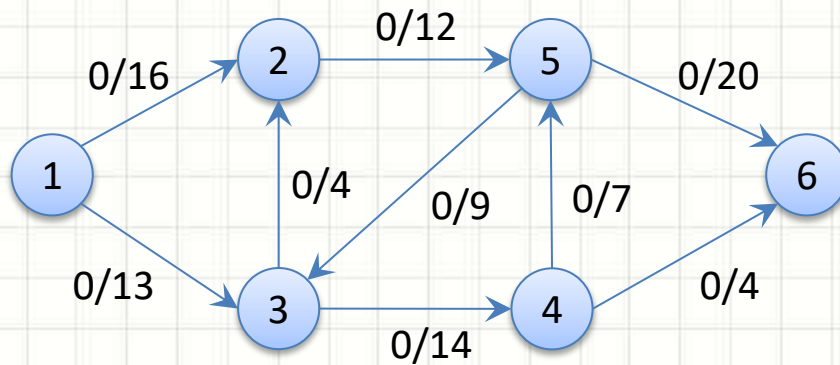


Método de Ford-Fulkerson

- Método com Complexidade $O(v+a)$
 - Limite em $O(a.f)$
- Passos
 1. Encontrar um caminho viável
 2. Descontar da capacidade
 3. Acrescentar fluxo reverso
 4. Volta para passo 1

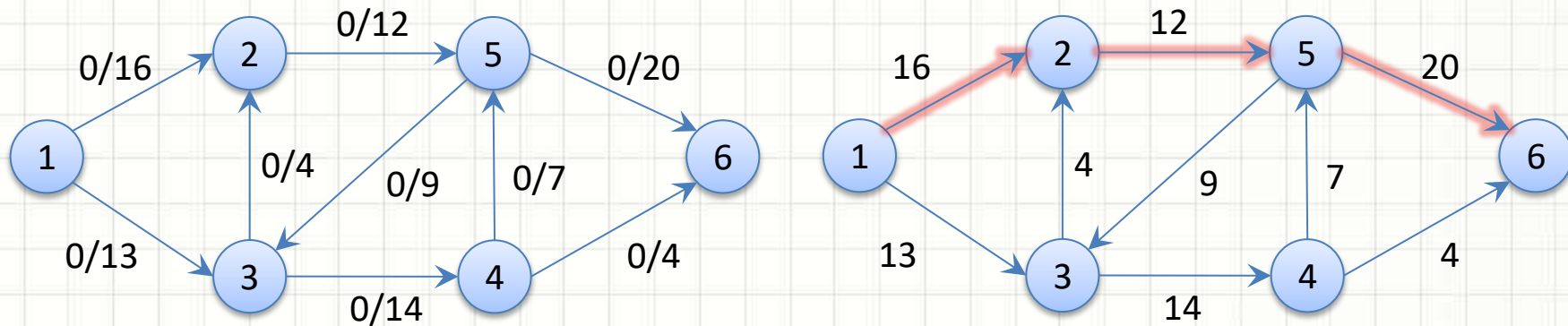
Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Encontrar caminho no residual



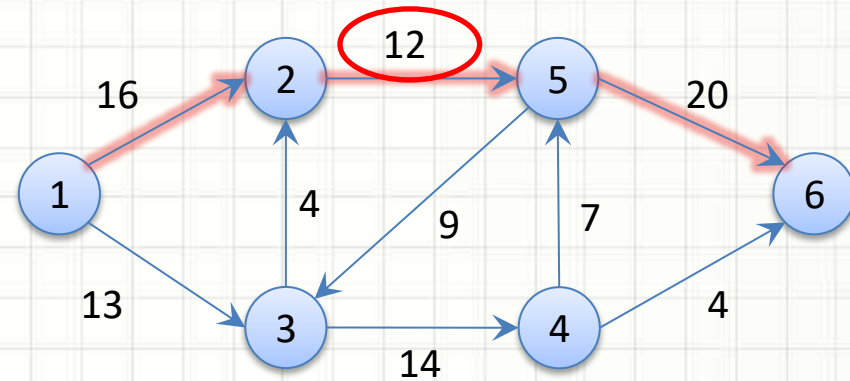
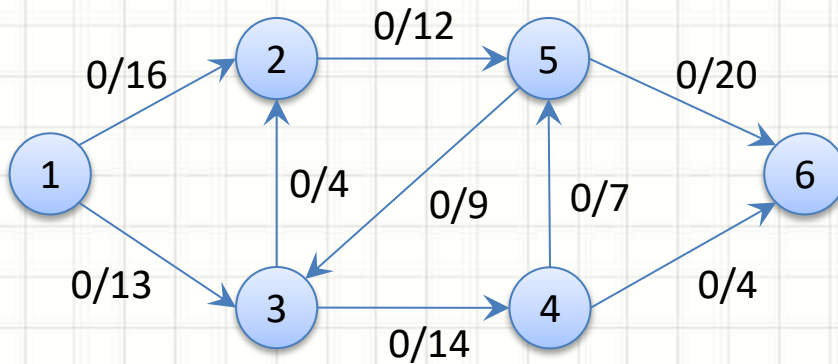
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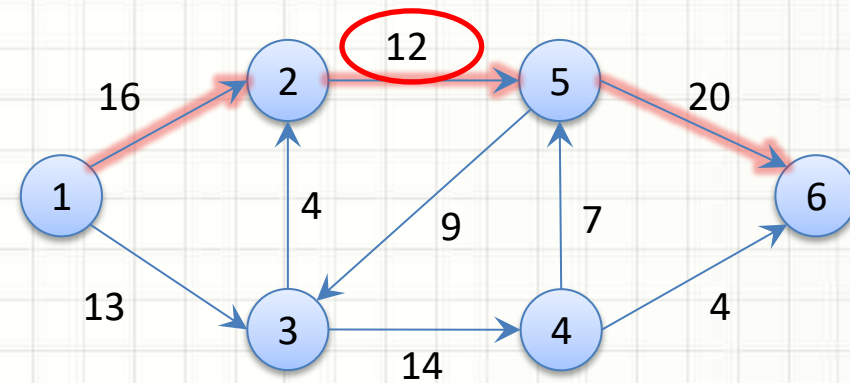
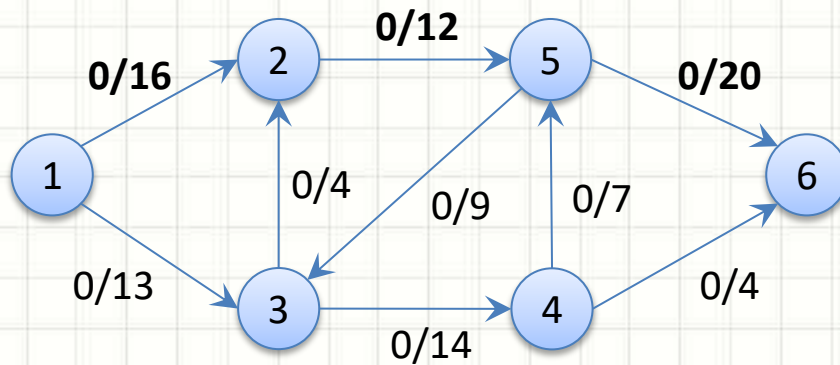
Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Identificar a menor capacidade do caminho



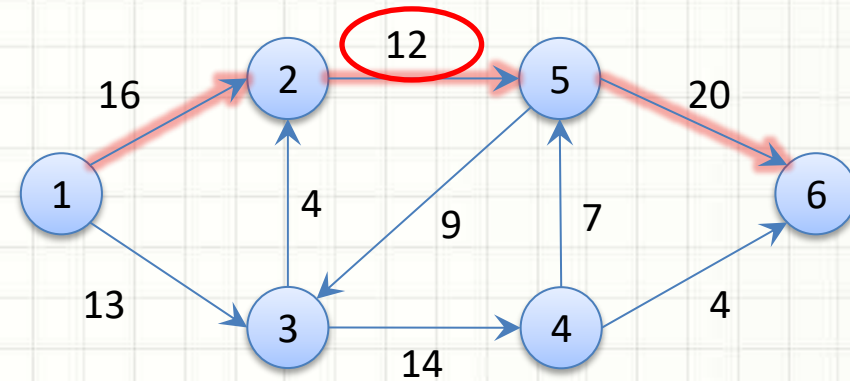
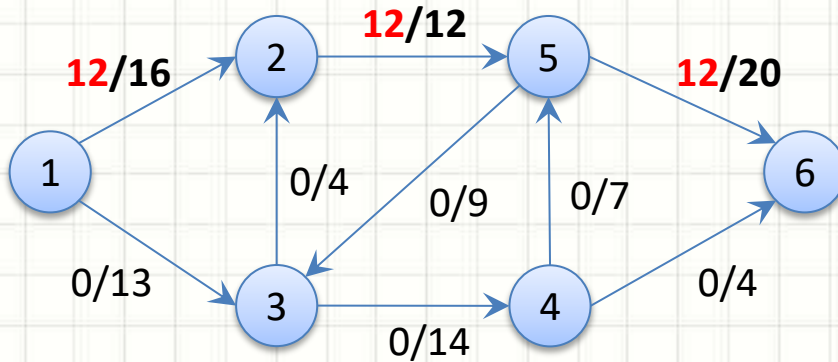
Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Alocar no grafo original



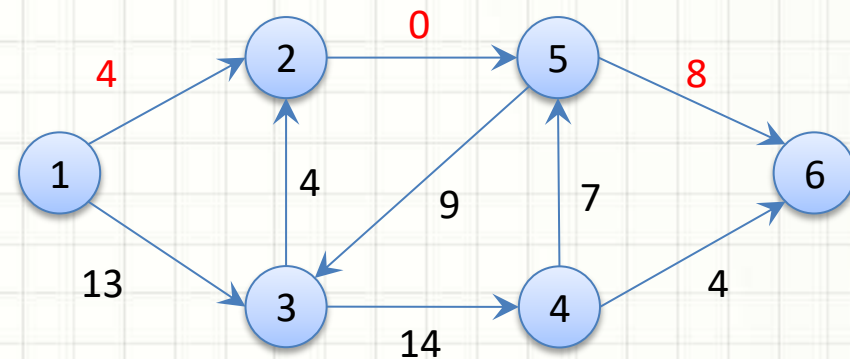
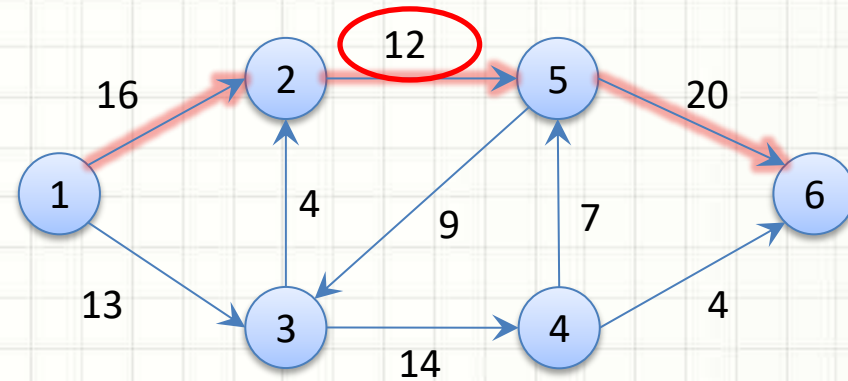
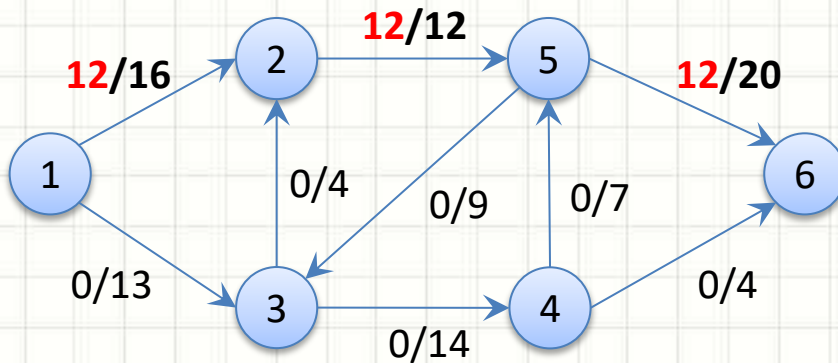
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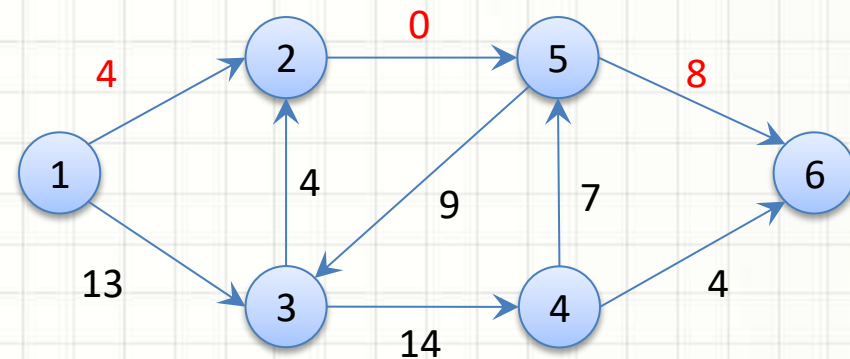
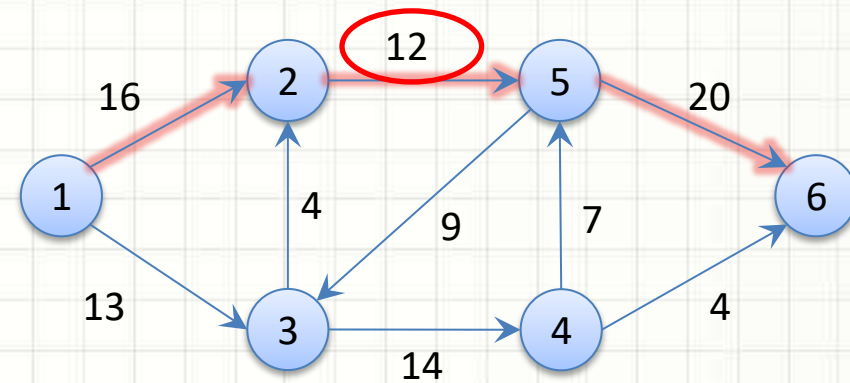
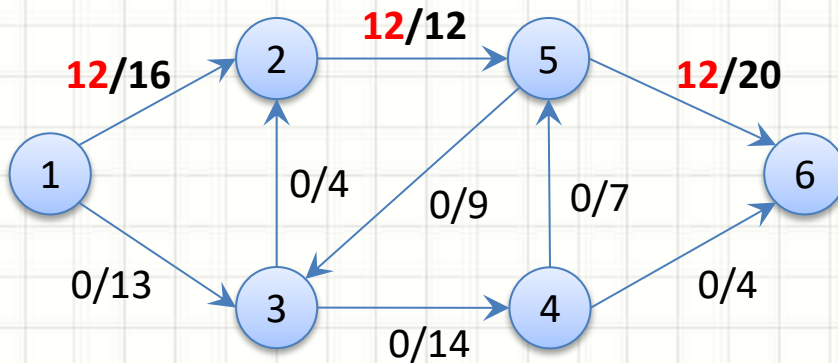
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- Grafo Base + Grafo Residual
 - Refazer o residual... **reduzindo a capacidade...**



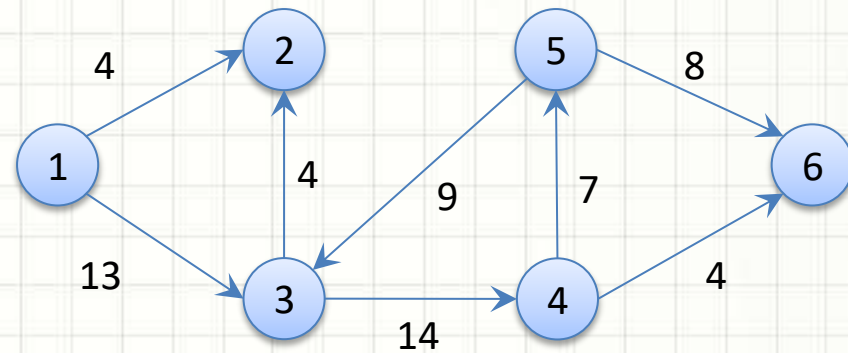
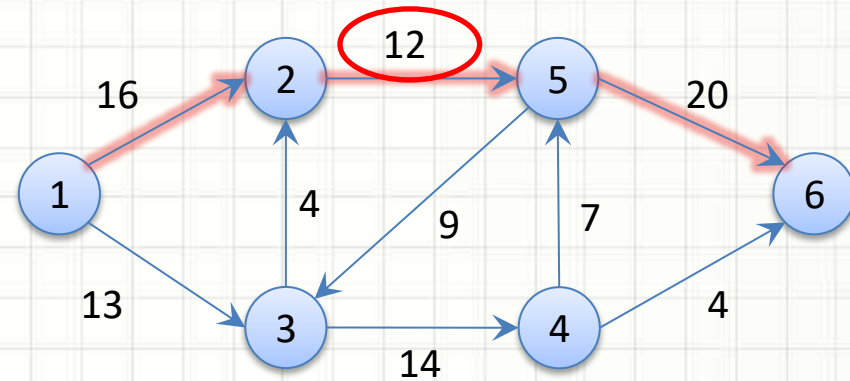
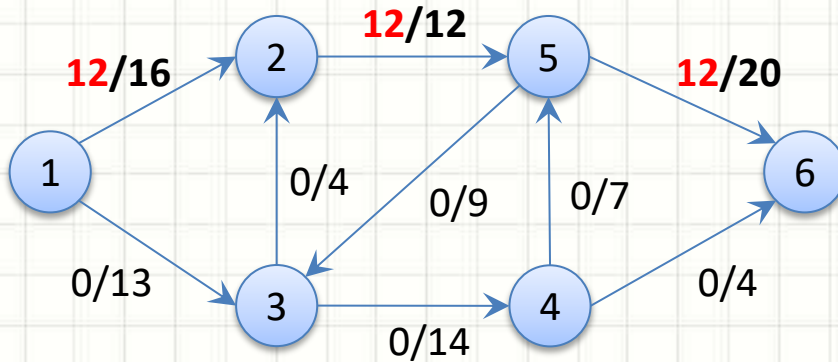
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 - Refazer o residual... **eliminando arcos zerados...**



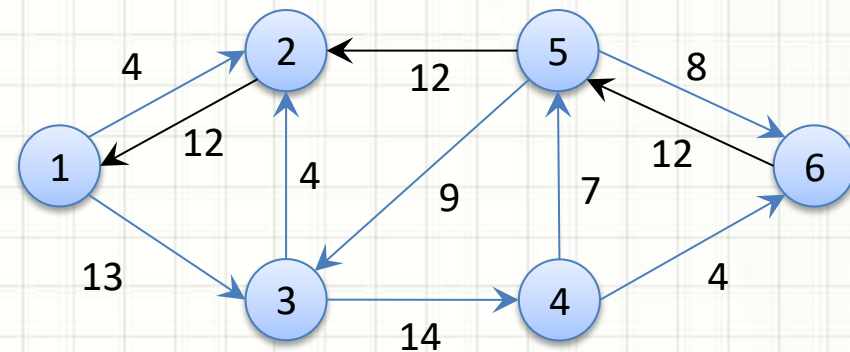
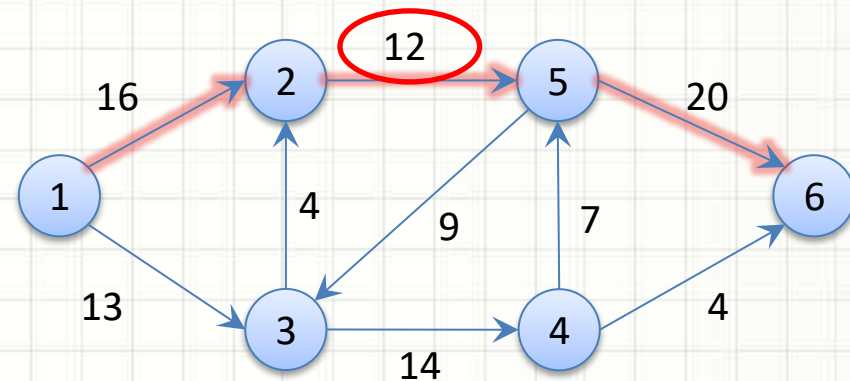
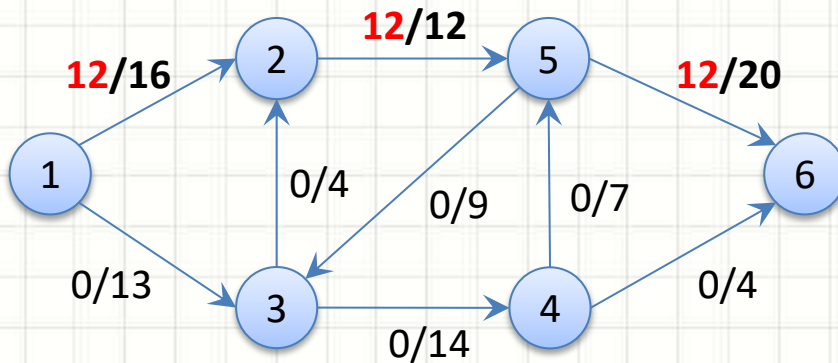
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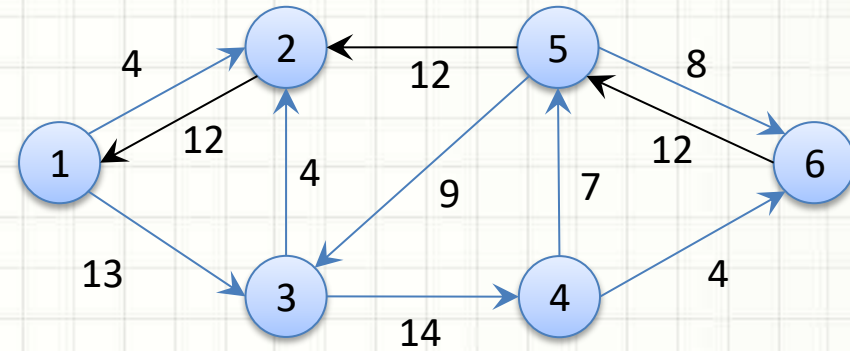
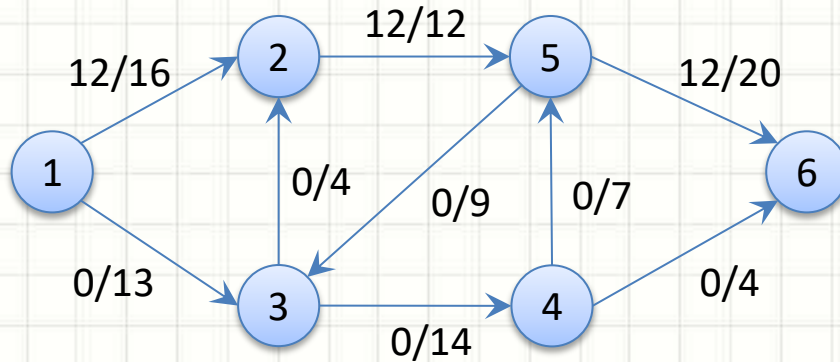
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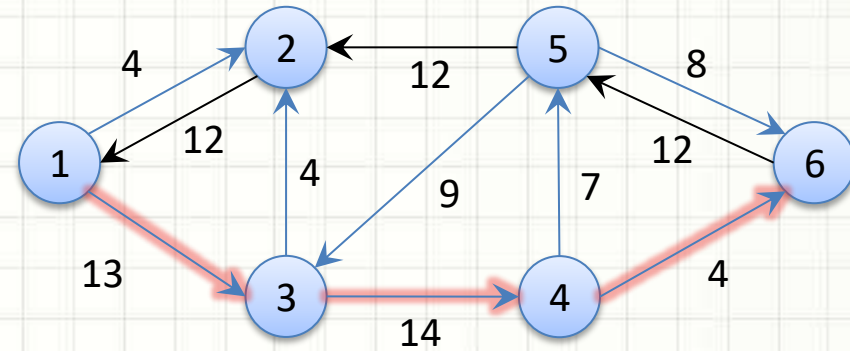
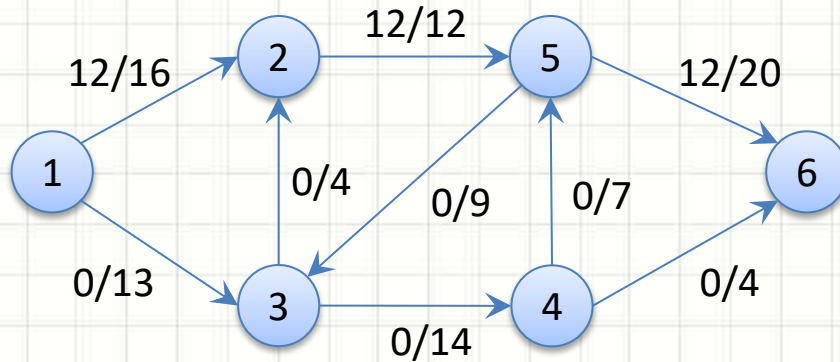
Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Encontrar novo caminho no residual...



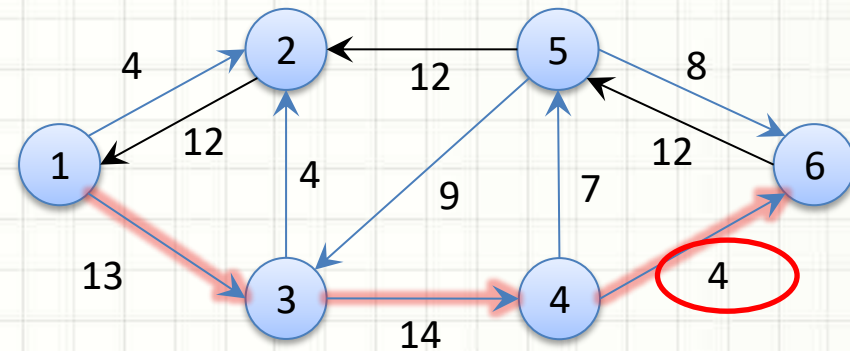
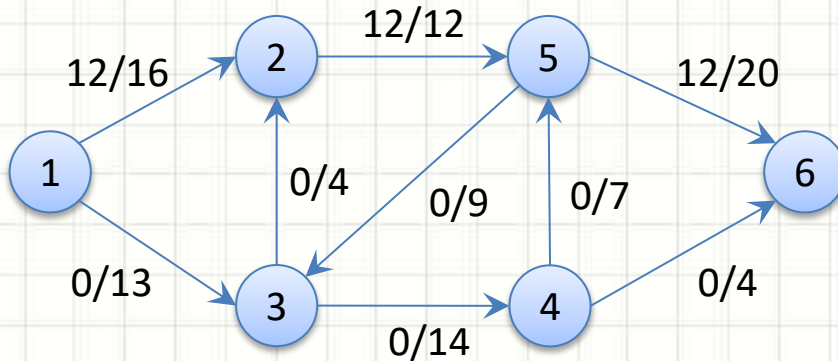
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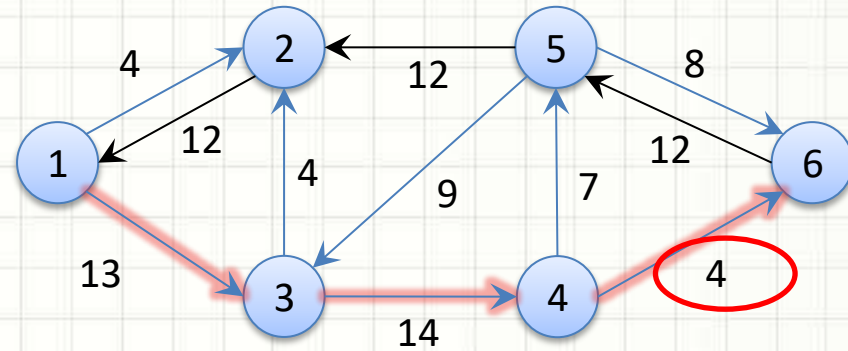
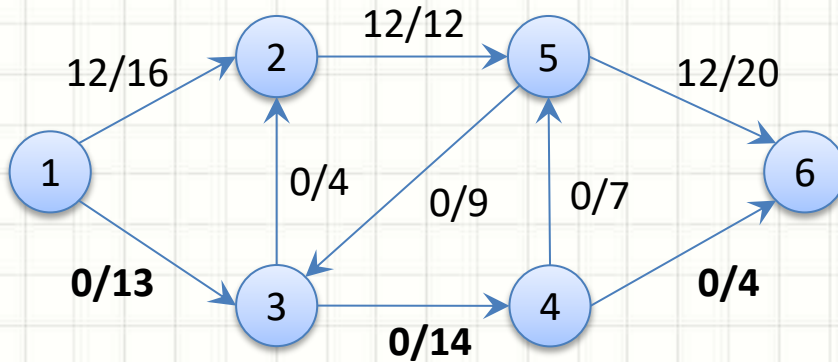
Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Identificar a menor capacidade do caminho



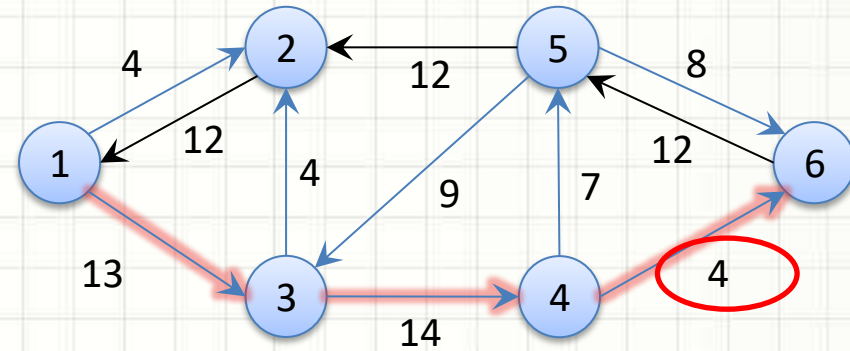
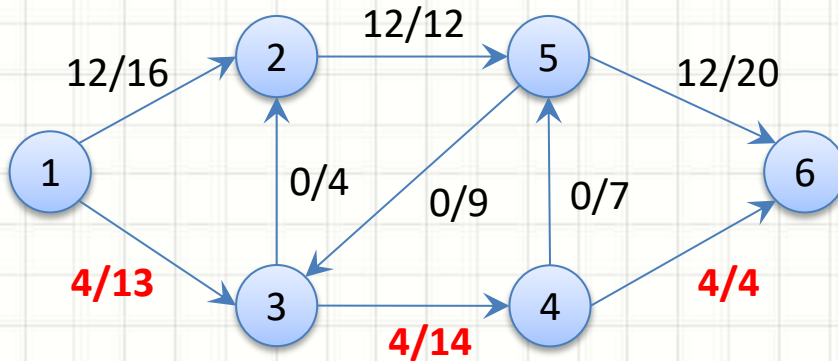
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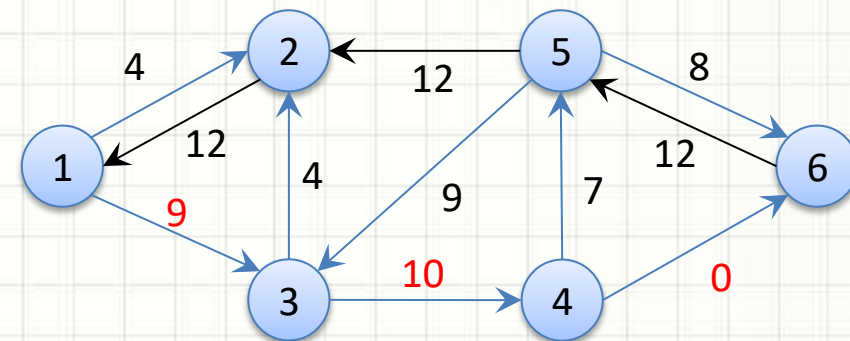
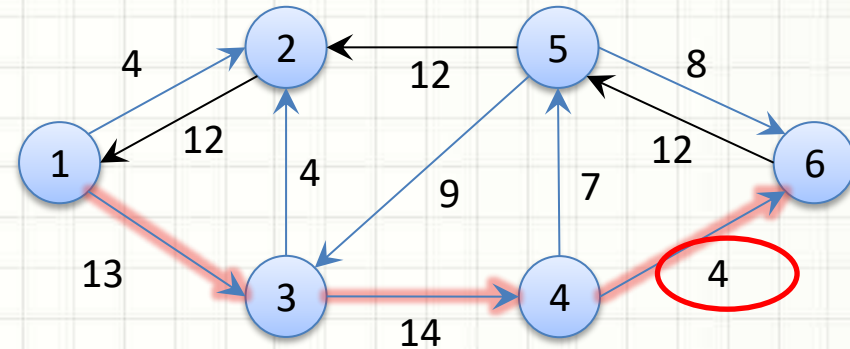
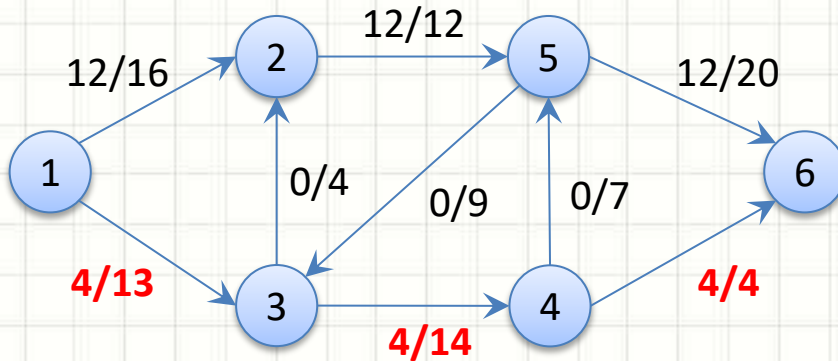
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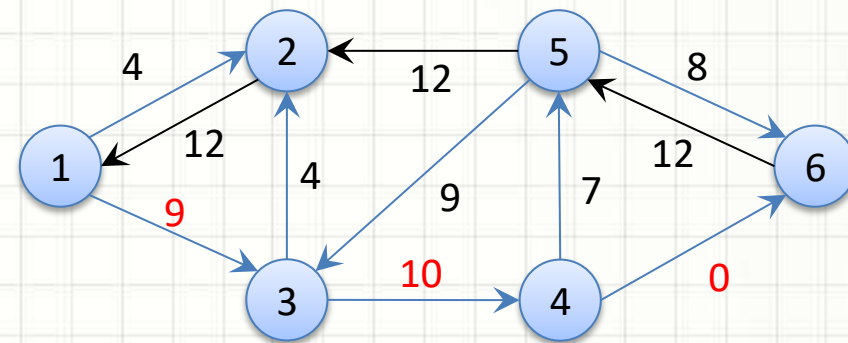
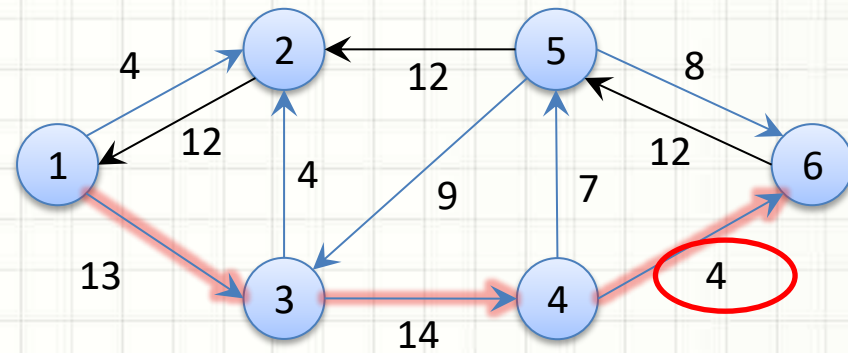
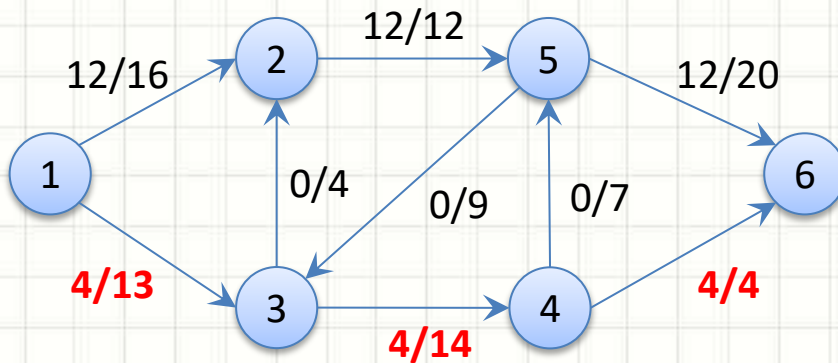
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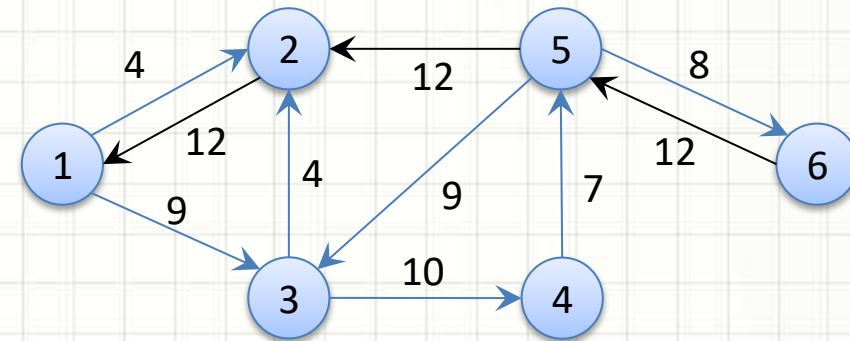
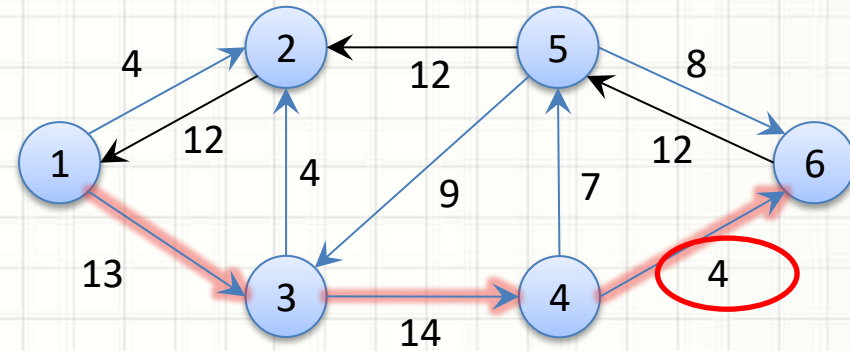
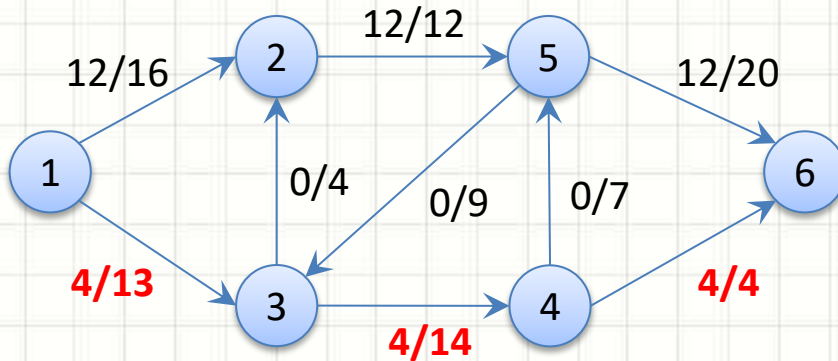
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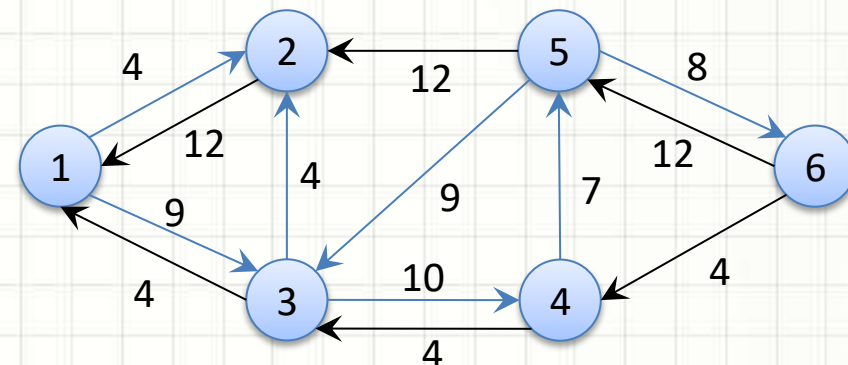
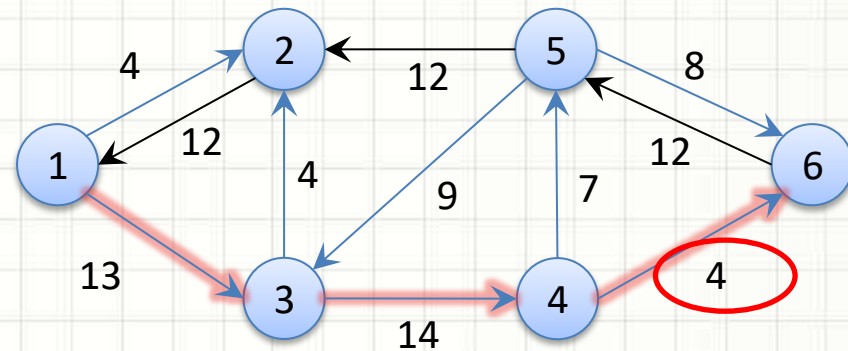
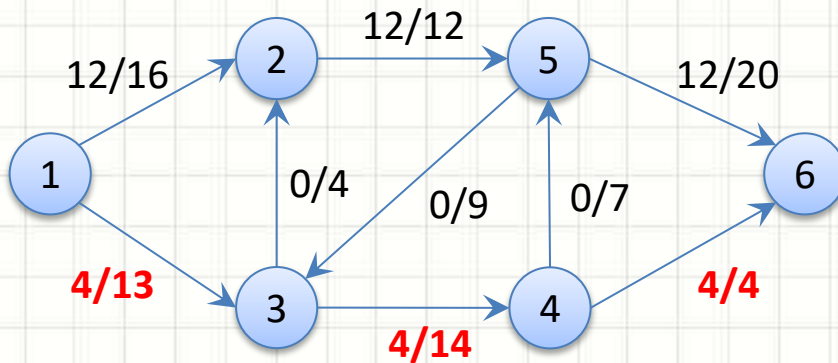
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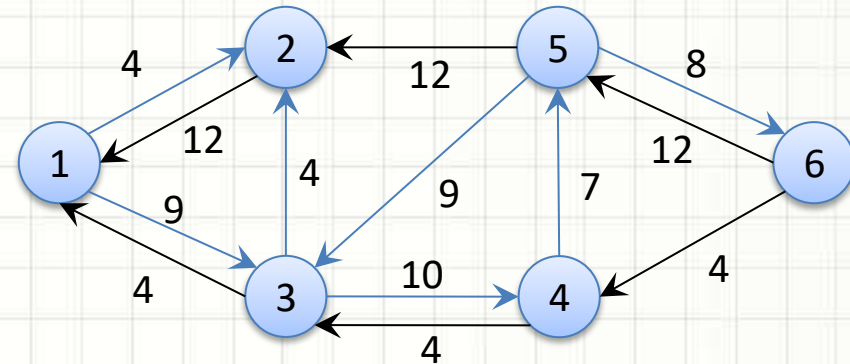
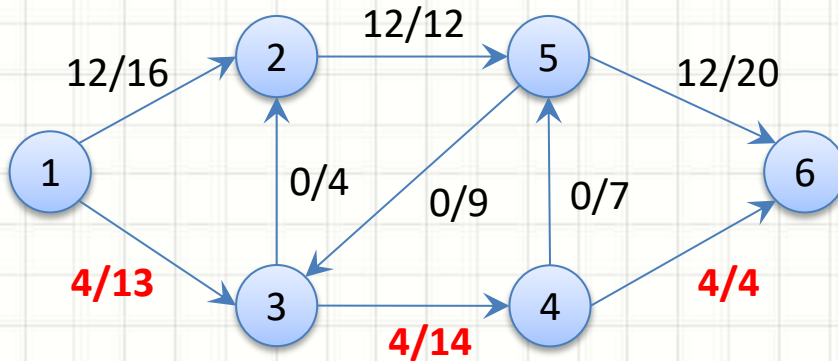
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 - Refazer o residual... **adicionando arcos reversos...**



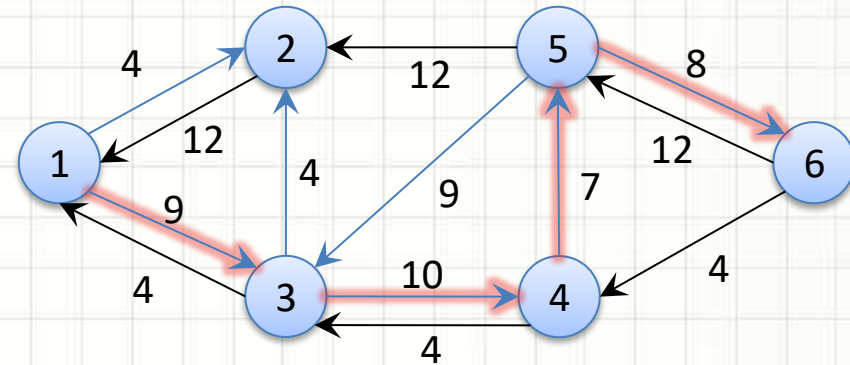
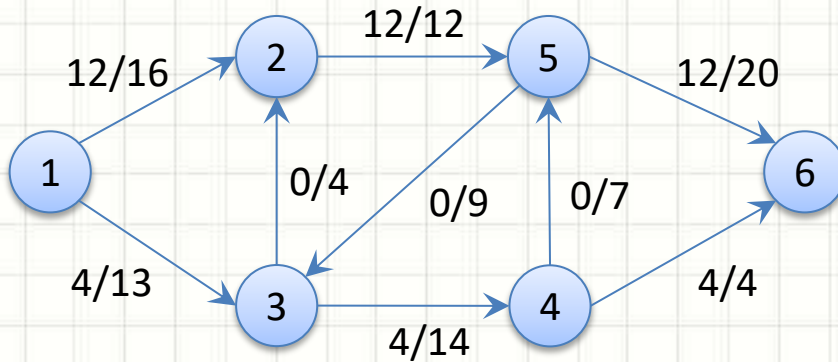
Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Encontrar novo caminho no residual...



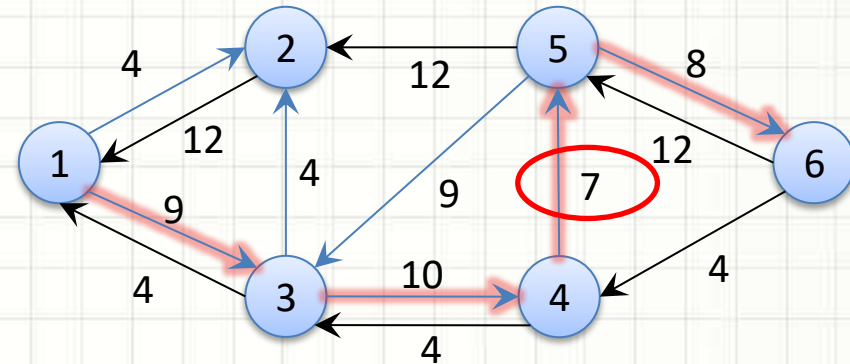
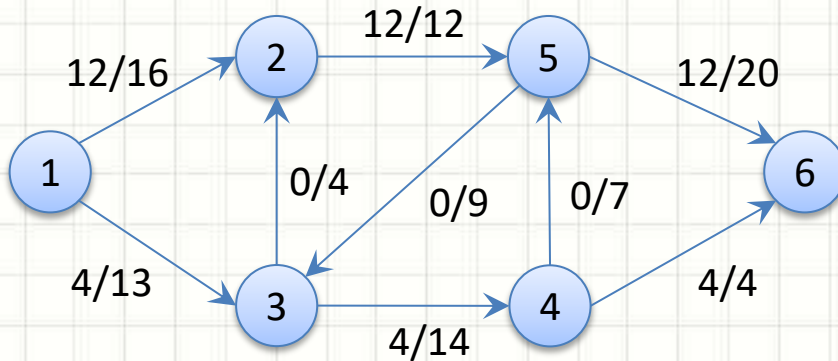
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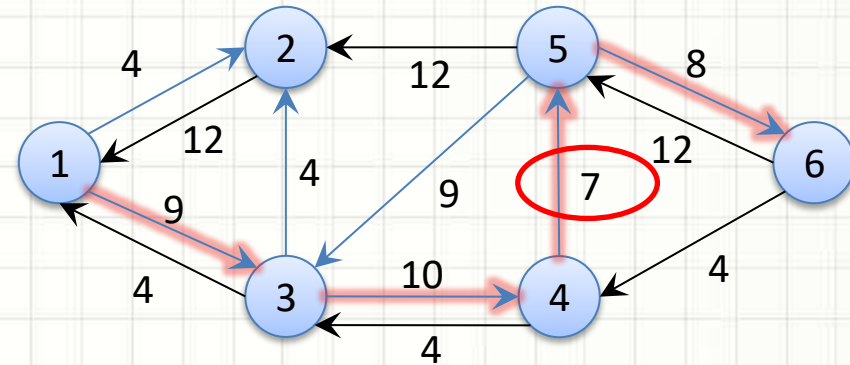
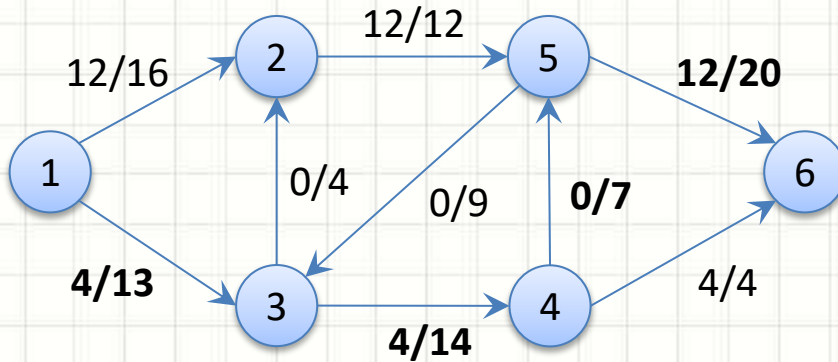
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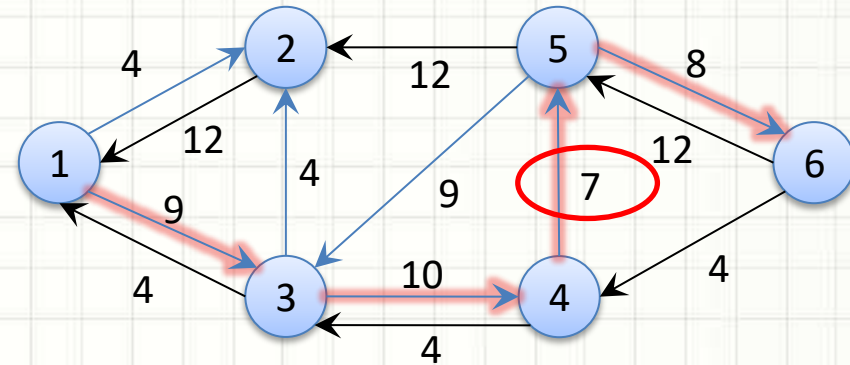
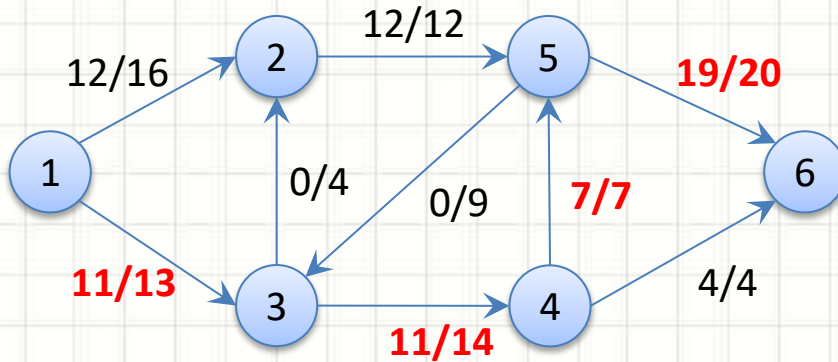
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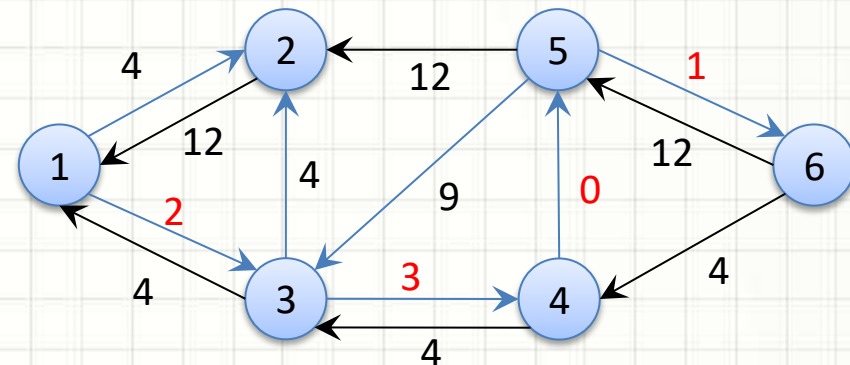
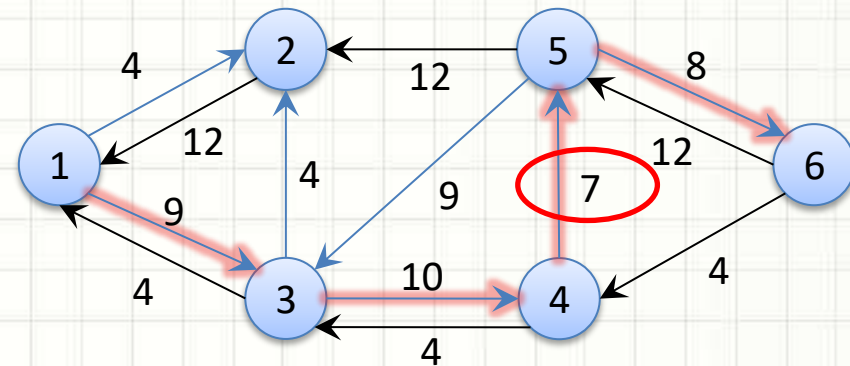
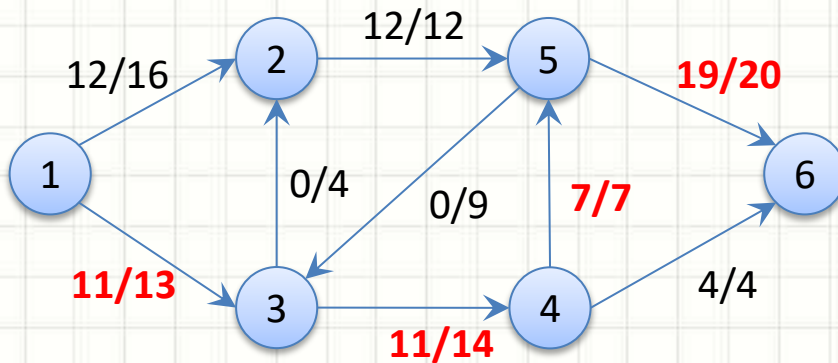
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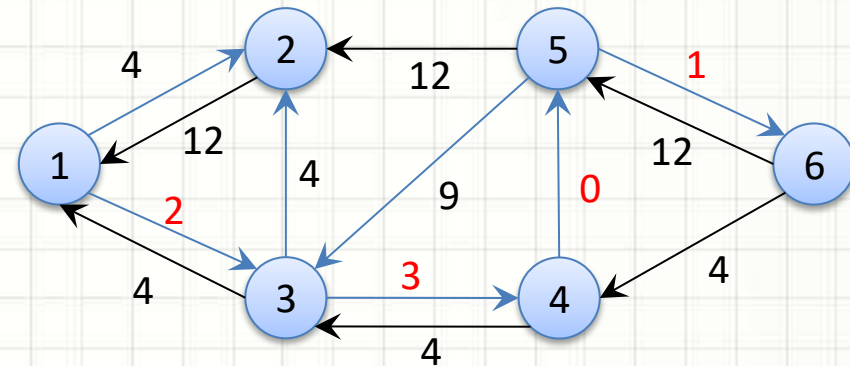
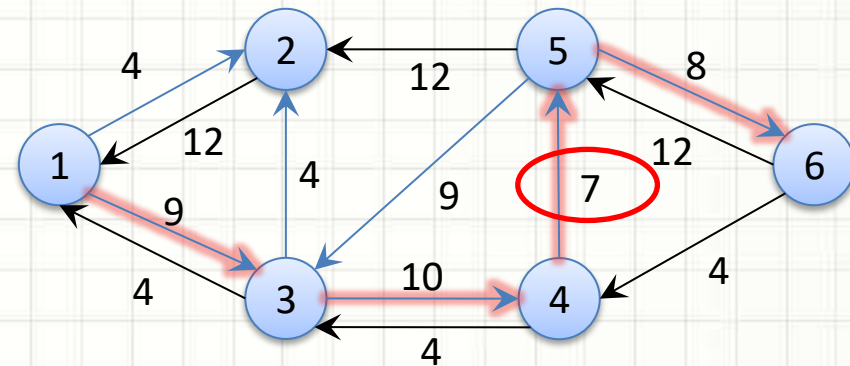
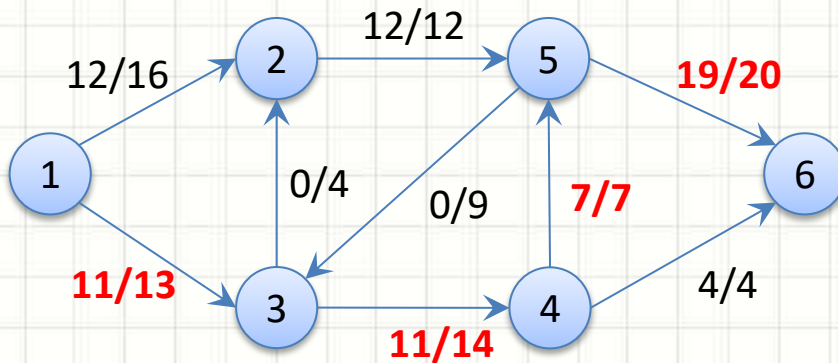
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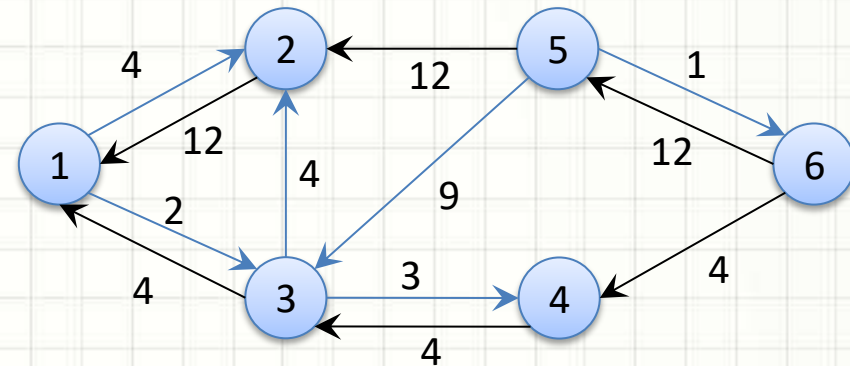
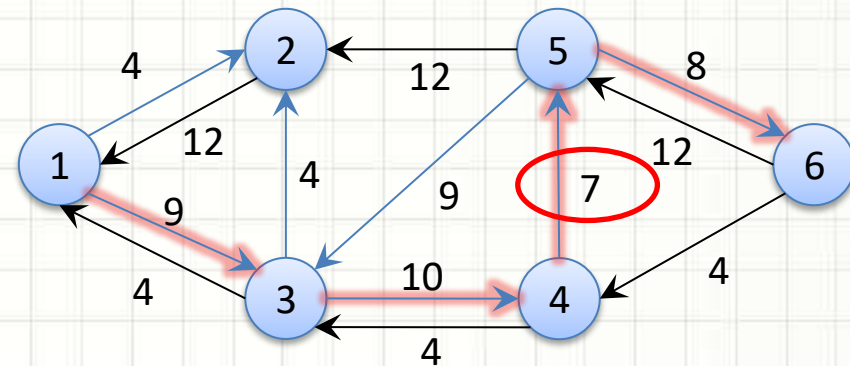
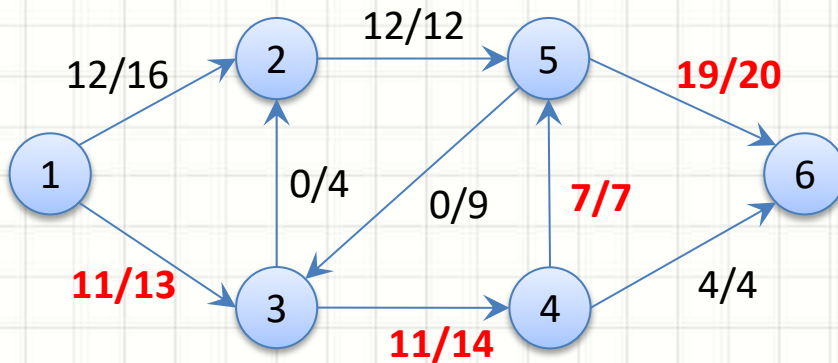
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 - Refazer o residual... **eliminando arcos zerados...**



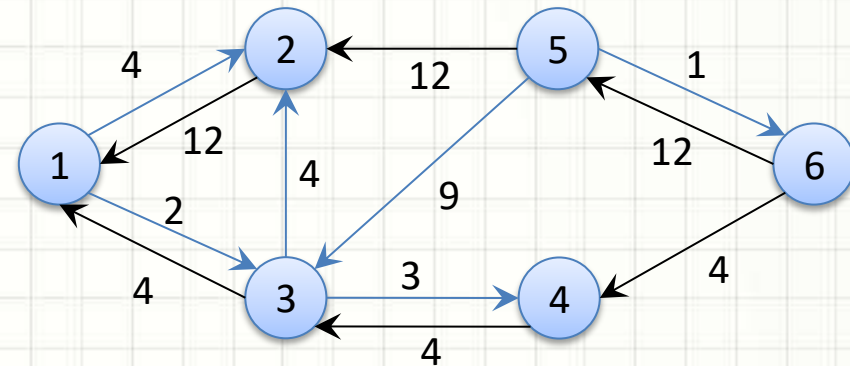
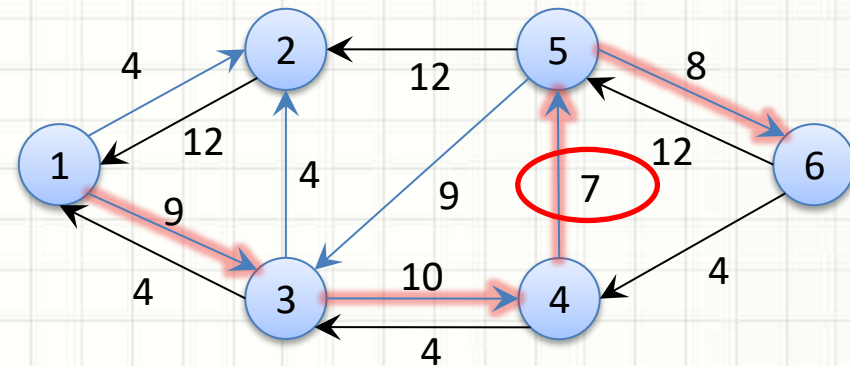
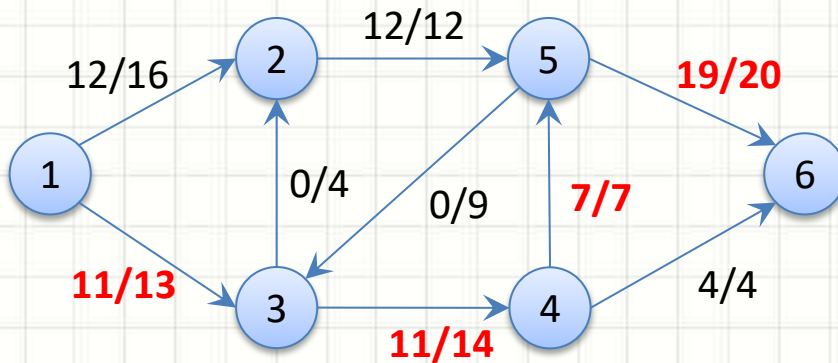
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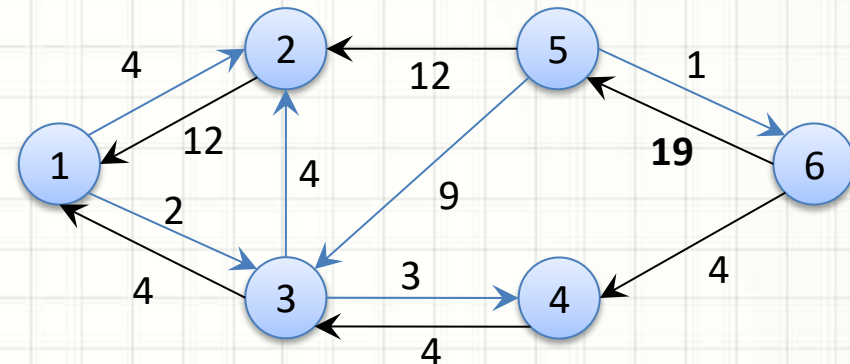
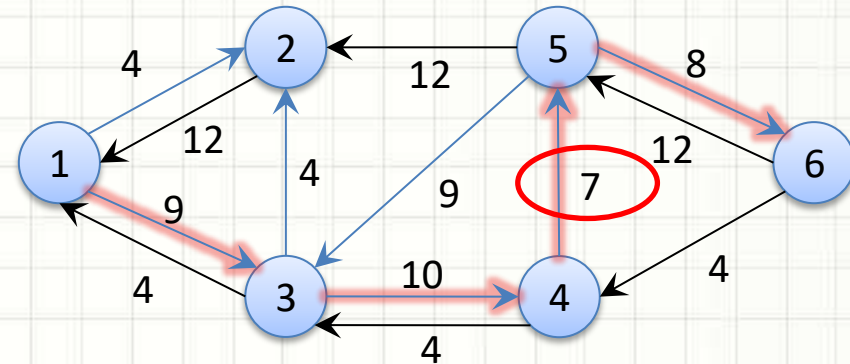
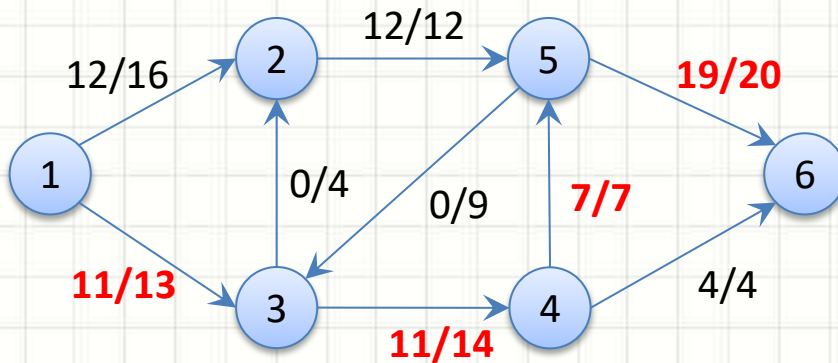
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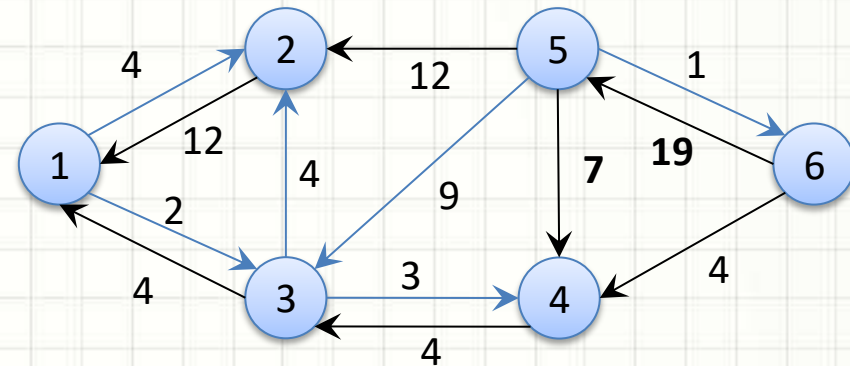
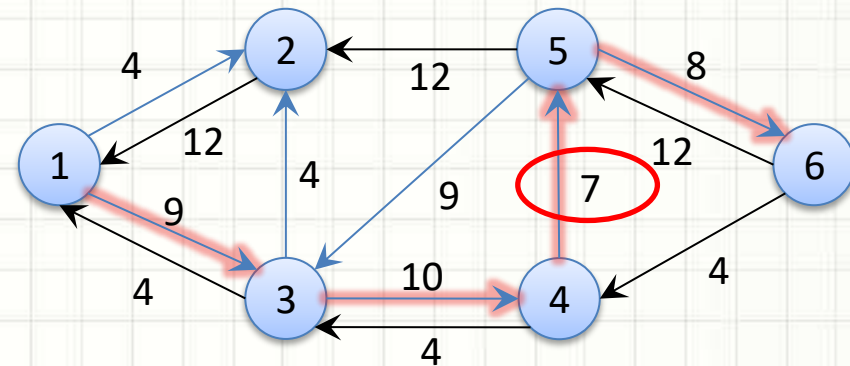
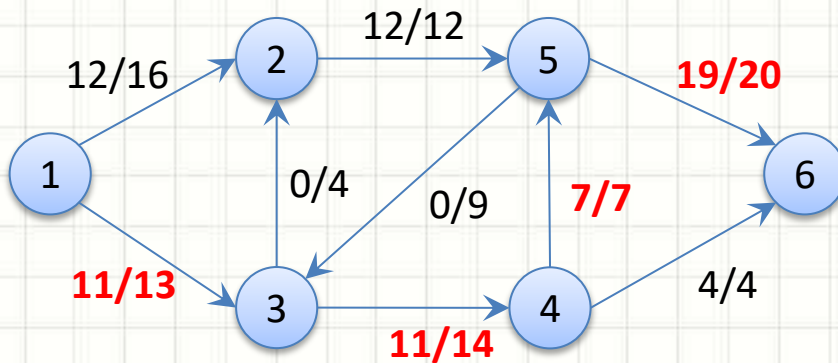
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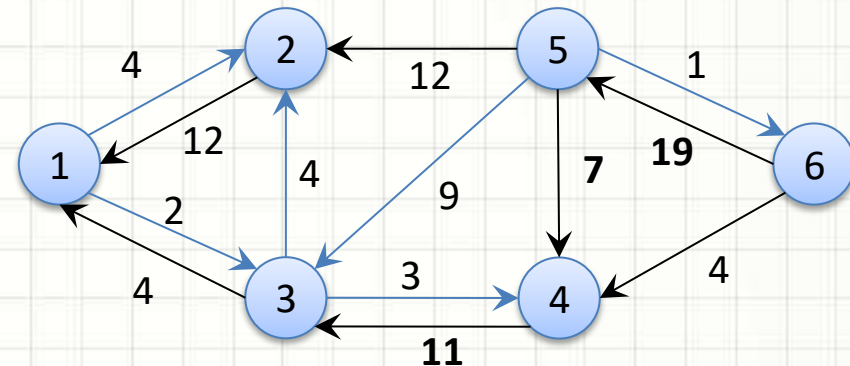
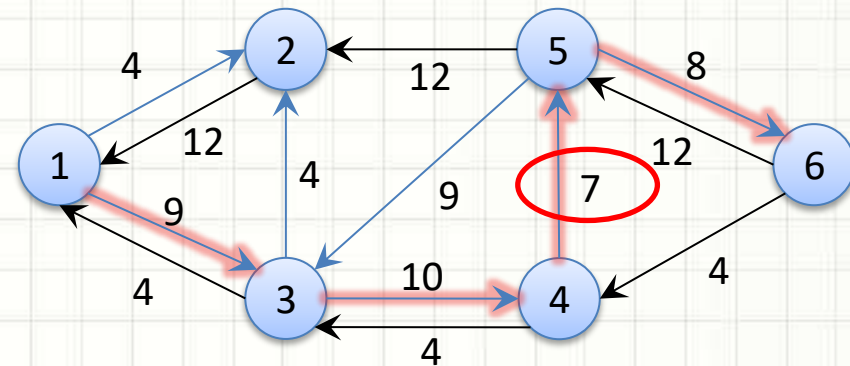
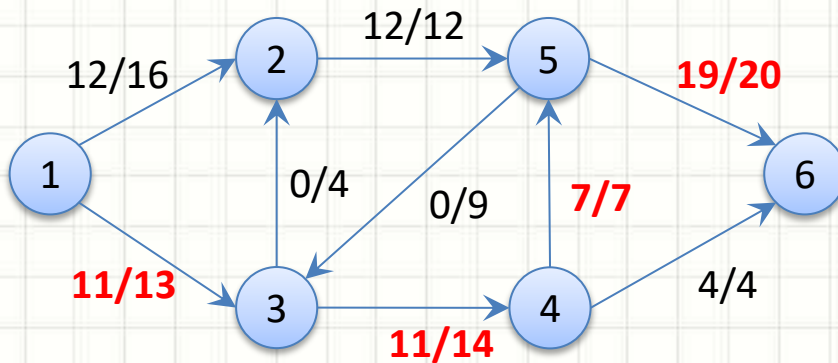
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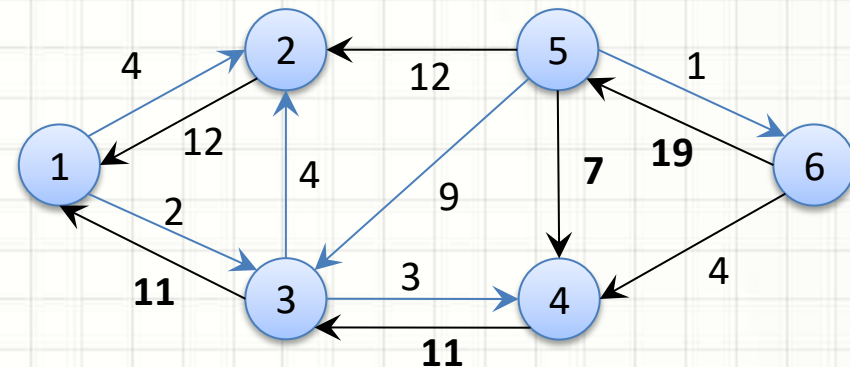
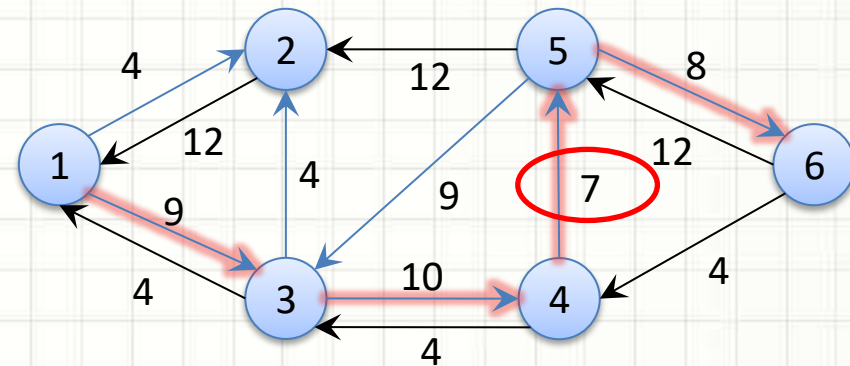
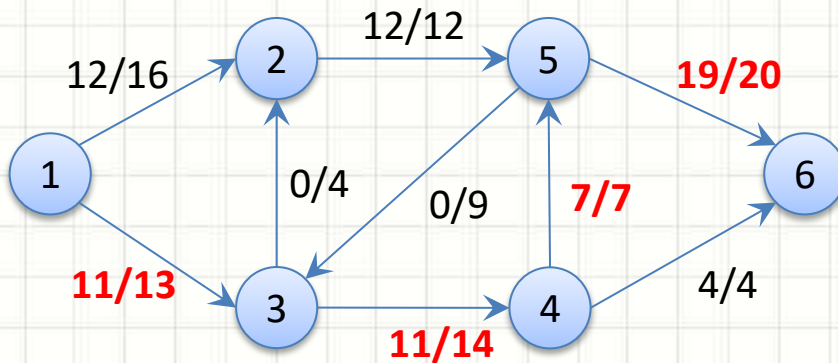
Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Refazer o residual... **adicionando arcos reversos...**



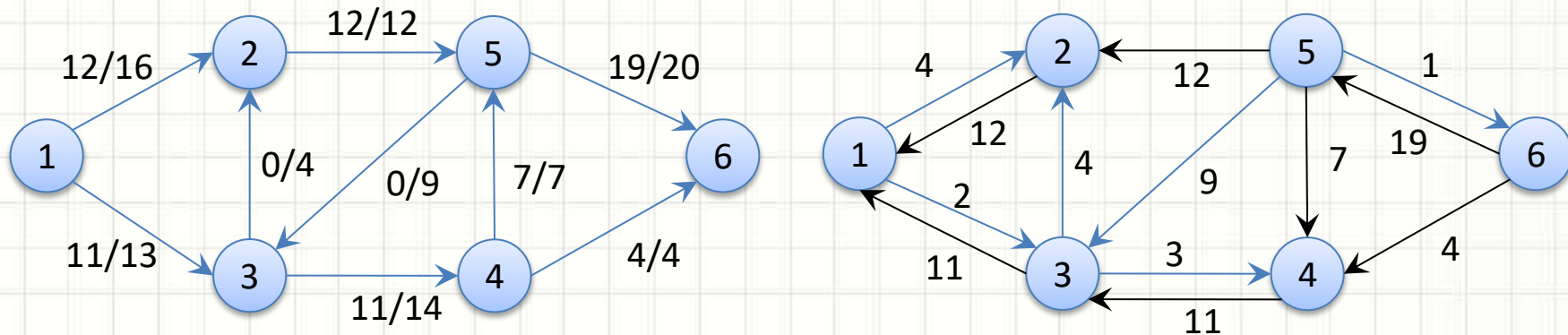
Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Refazer o residual... **adicionando arcos reversos...**



Método de Ford-Fulkerson

- Grafo Base + Grafo Residual
 - Encontrar novo caminho no residual...

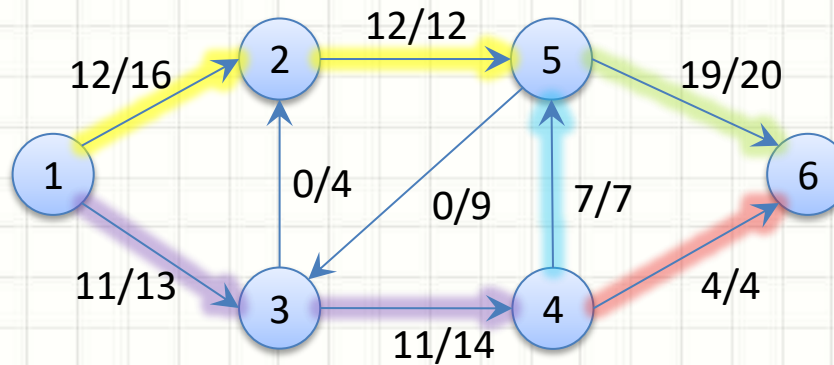


**Como não há caminho viável,
chegamos no ótimo!**

O Fluxo Máximo é 23

Método de Ford-Fulkerson

- Caminhos Encontrados

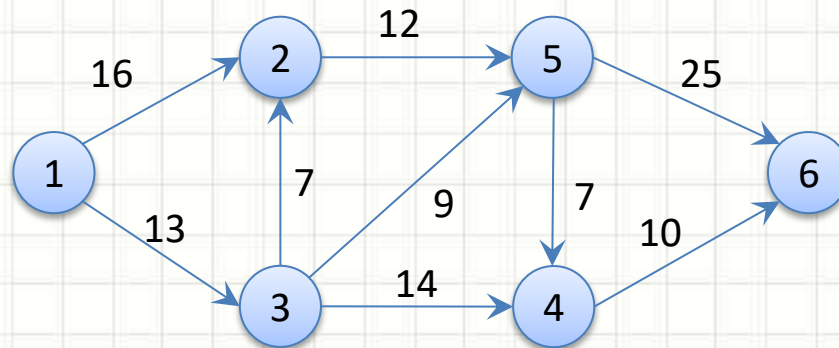




EXEMPLO E EXERCÍCIO

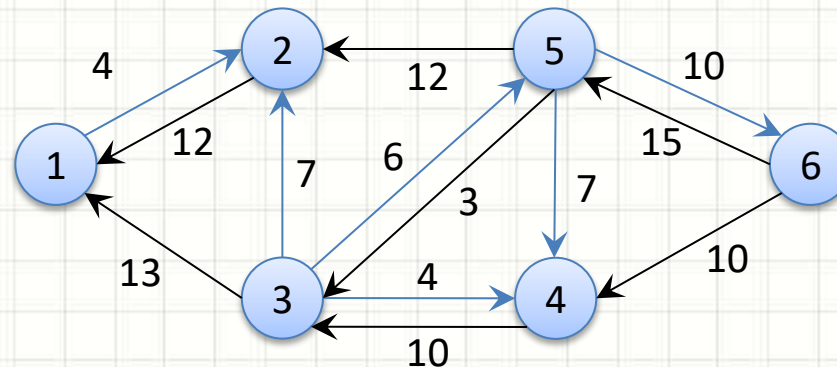
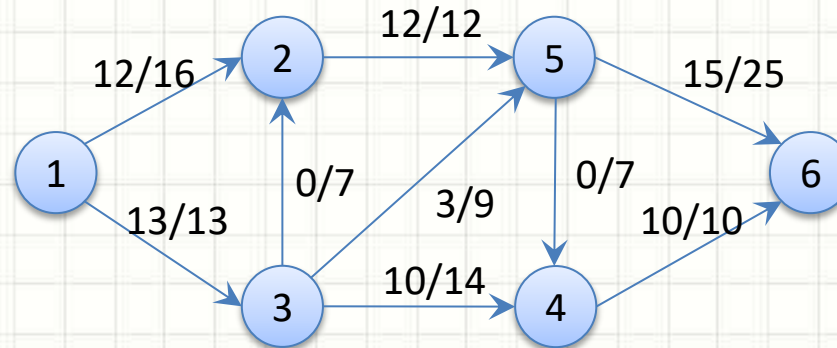
Exemplo

- Aplique o método de Ford-Fulkerson



Exemplo

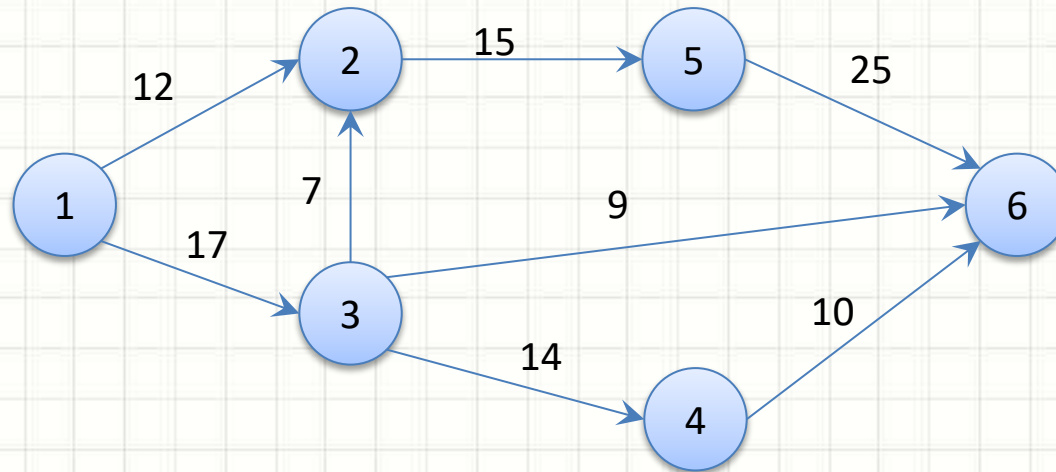
- Aplique o método de Ford-Fulkerson



O Fluxo Máximo é 25

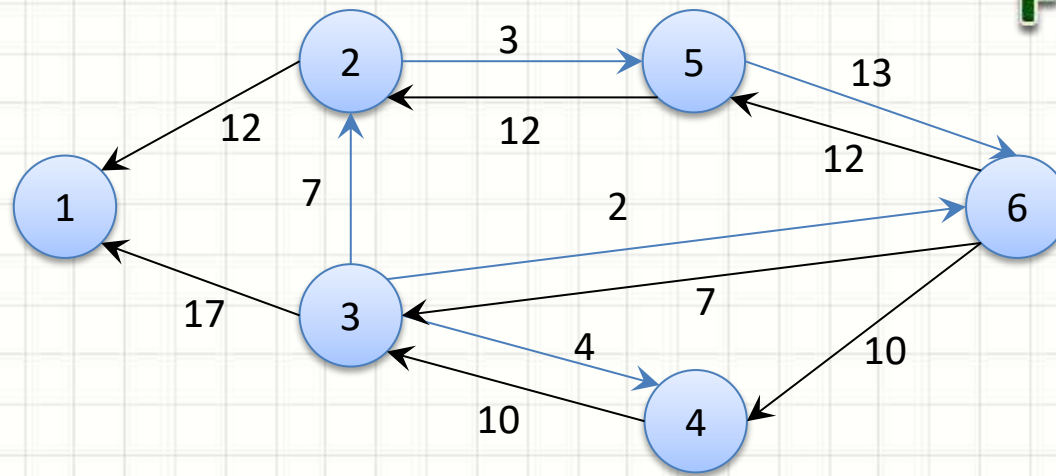
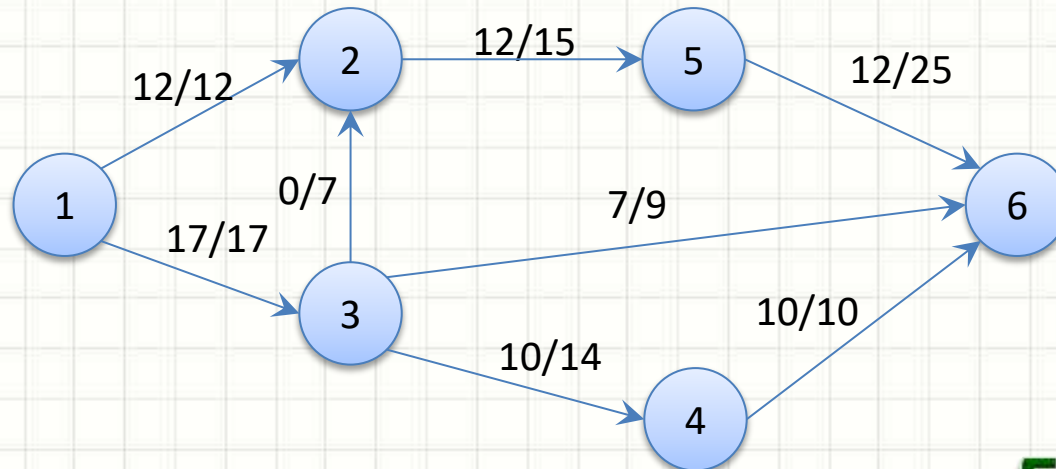
Exercício

- Aplique o método de Ford-Fulkerson



Exercício

- Aplique o método de Ford-Fulkerson



$F_{\text{máx}}$ é 29



CONCLUSÕES

Resumo

- Fluxo Máximo de uma Rede
 - Importante em problemas de escoamento
 - Método de Ford-Fulkerson
 - **TAREFA:** Exercícios Aula 10
-

- Problema do Caixeiro Viajante
 - O problema da cobertura dos nós



PERGUNTAS?



EXERCÍCIO

Exercício (para casa)

- Identifique o fluxo máximo

