



# **MECÂNICA DOS SÓLIDOS**

## **TRELIÇAS PLANAS**

### **PARTE I**

Prof. Dr. Daniel Caetano

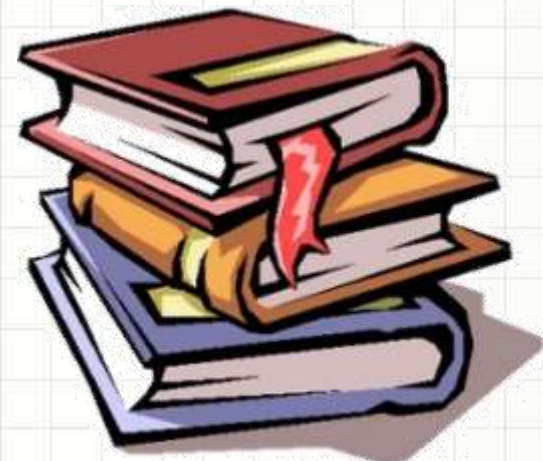
2019 - 1

# Objetivos

- Aplicar os conceitos vistos anteriormente
  - Conhecer a geometria de uma treliça
  - Compreender como os esforços agem em uma treliça e seu comportamento
  - Modelagem computacional com FTool
- 
- **Atividade Aula 4 – SAVA!**
  - **Pré-Aula 05 – SAVA**



# Material de Estudo



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## Material

## Acesso ao Material

Apresentação

<http://www.caetano.eng.br/>  
(Mecânica dos Sólidos – Aula 4)

Material Didático

Mecânica Geral (MACIEL), Cap. 3.2 (SAVA)

Minha Biblioteca

Estática e Mecânica dos Materiais (BERR;JOHNSTON),  
Cap. 6 – Seção 6.1 a 6.6

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**LEMBRETE: CONSULTAR O “ANTES” DA AULA 5 NO SAVA!**



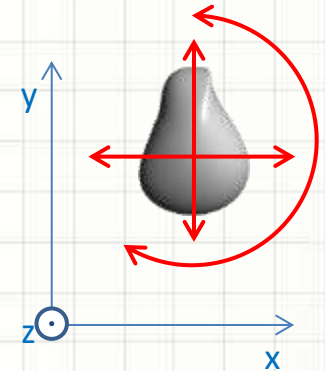
**RELEMBRANDO:**

**EQUILÍBRIO DE**

**CORPOS RÍGIDOS**

# Graus de Liberdade e Equilíbrio

- No plano, corpo rígido: 3 graus de liberdade
  - Translação horizontal
  - Translação vertical
  - Rotação no plano (ao redor de z)



- Condição de Equilíbrio do Corpo Rígido

**Condição de  
Equilíbrio em X**

$$R_x = \sum F_x = 0$$

**Condição de  
Equilíbrio em Y**

$$R_y = \sum F_y = 0$$

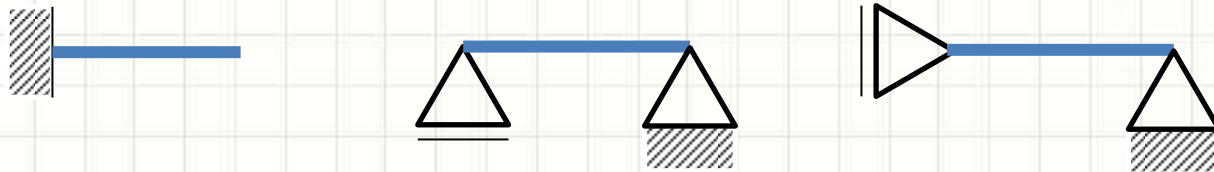
**Condição de  
Equilíbrio de Momentos**

$$M_{R0} = \sum M_o = 0$$

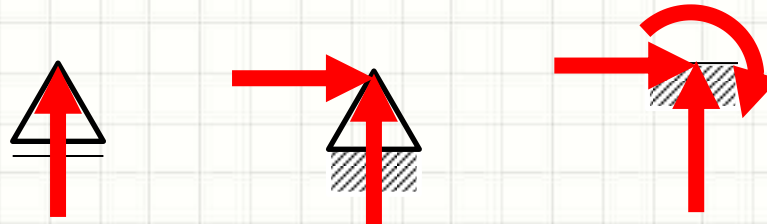


# Vínculos e Reações de Apoio

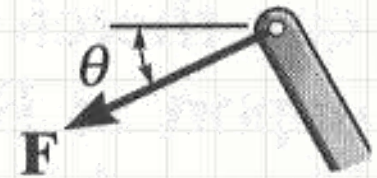
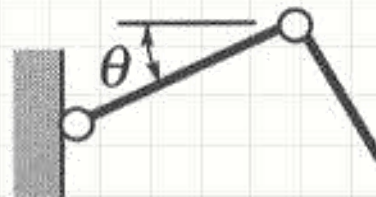
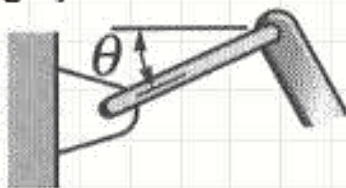
- Equilíbrio estático
  - Todos os graus de liberdade impedidos



- Cada tipo **pode** impor diferentes reações



Ligação esbelta

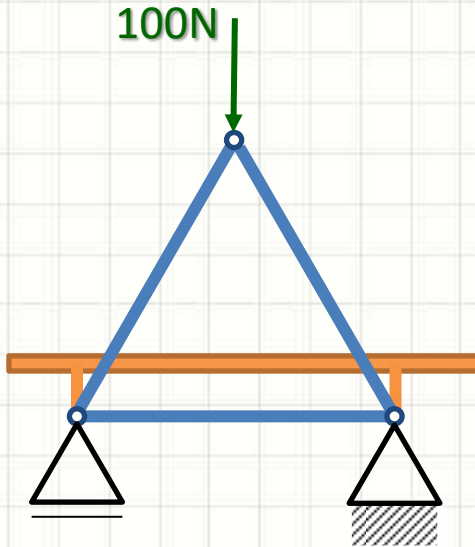


The background features a light gray grid pattern. Overlaid on the top left is a decorative graphic consisting of several overlapping, wavy, semi-transparent red lines that curve across the top of the page. A dashed red line also follows a similar curved path, positioned slightly below the solid red lines.

# TRELIÇAS PLANAS RÍGIDAS

# Treliças Planas Rígidas

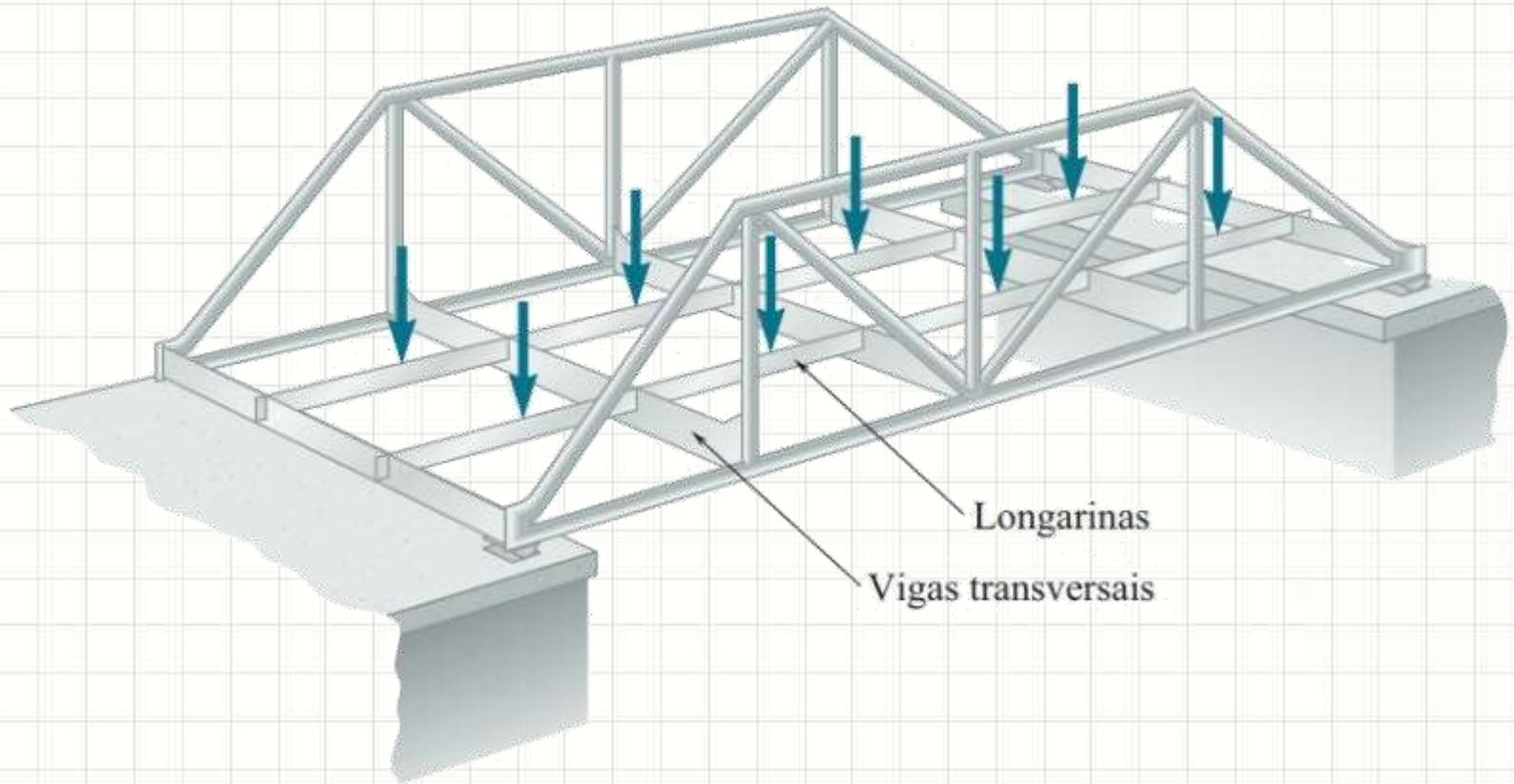
- Estrutura composta por barras
  - Em geral, compondo elementos triangulares
  - Articuladas nas extremidades
  - As cargas são aplicadas sempre nos nós



**E o peso das barras?**

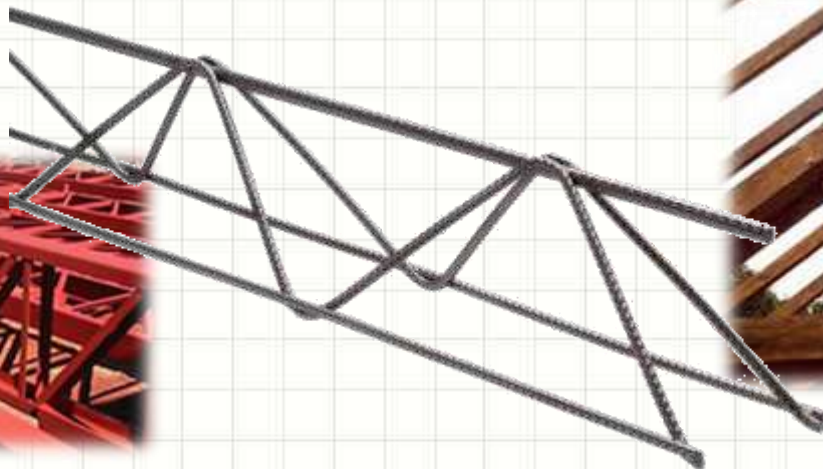


# Exemplo de Aplicação: Ponte



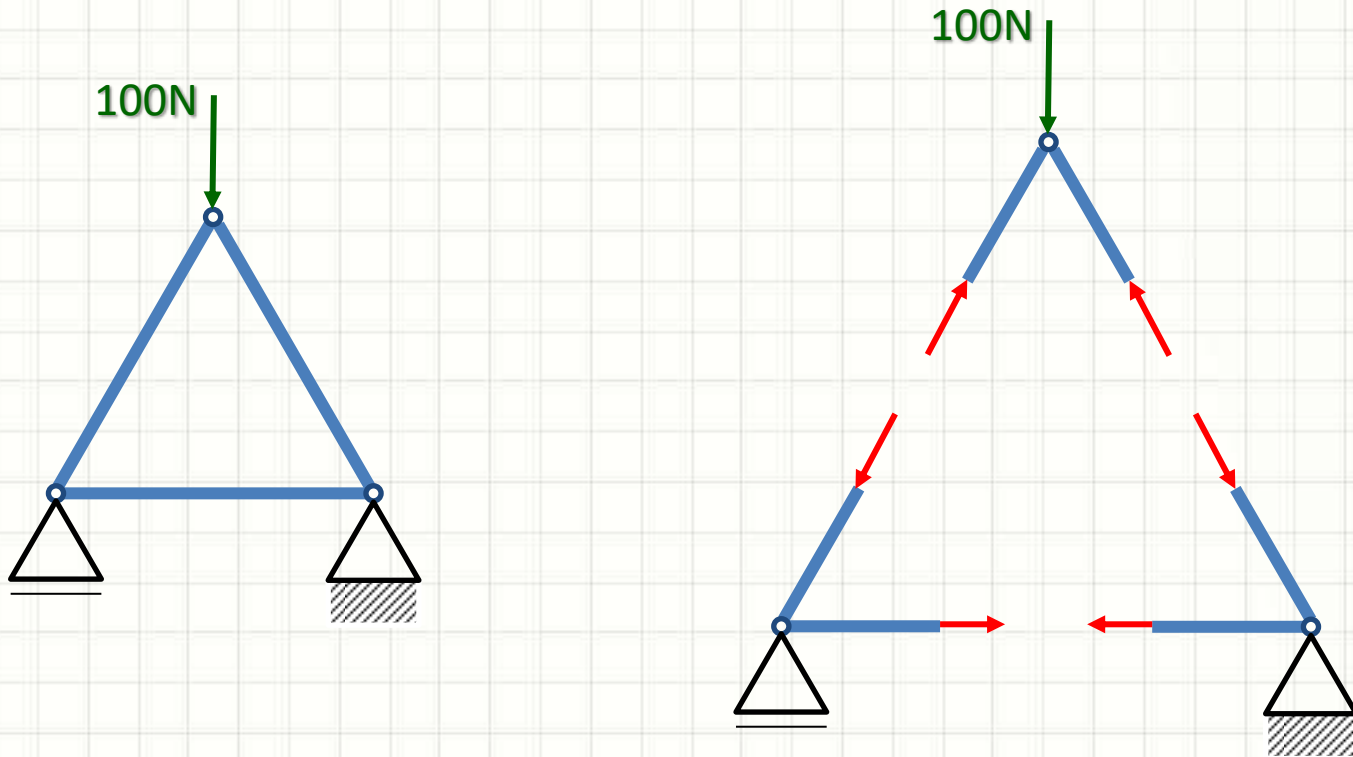
# Treliças Planas Rígidas

- Na prática...
  - As extremidades nem sempre são articuladas



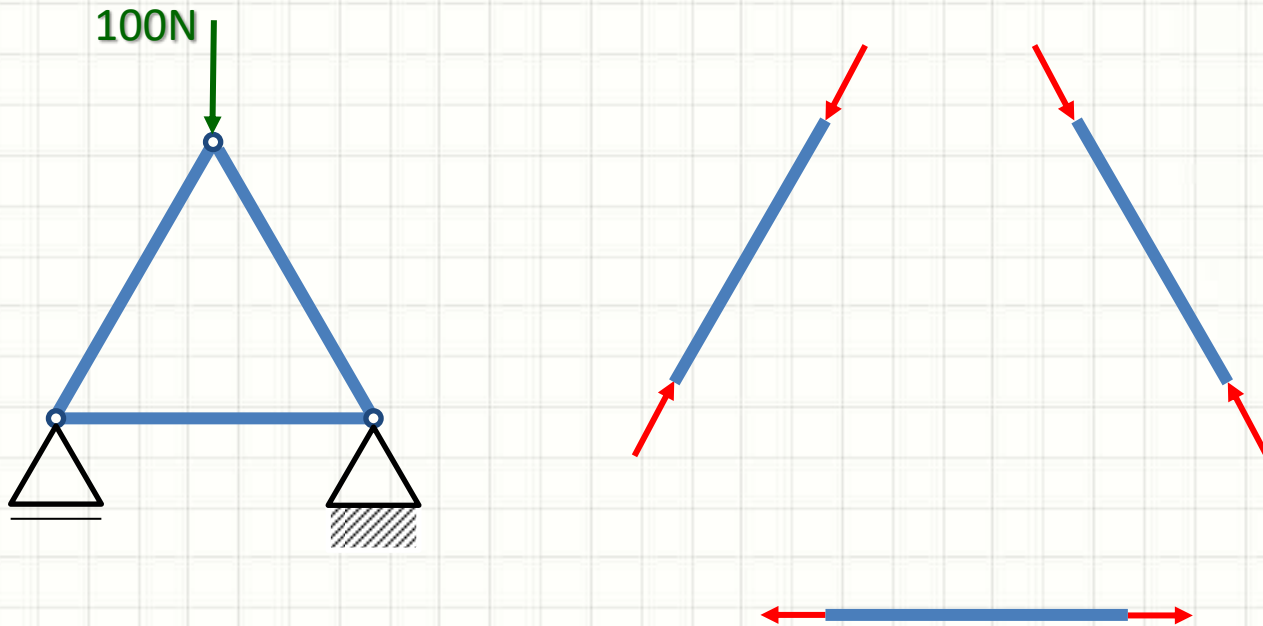
# Forças Internas

- Até agora... Forças Externas
  - Equilíbrio externo
- Forças Internas: mantém estrutura coesa



# Forças Internas

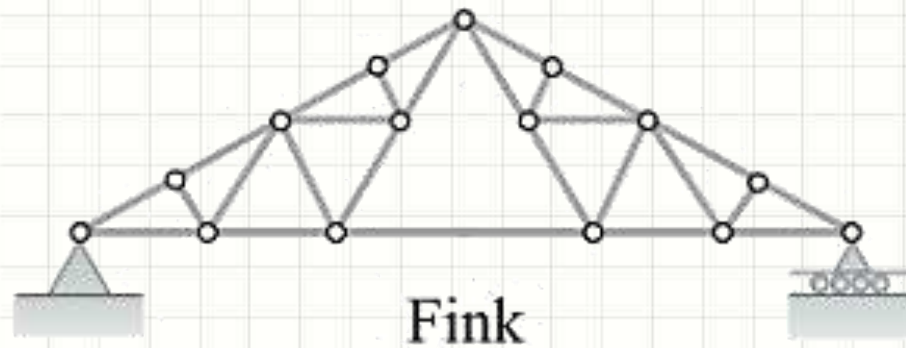
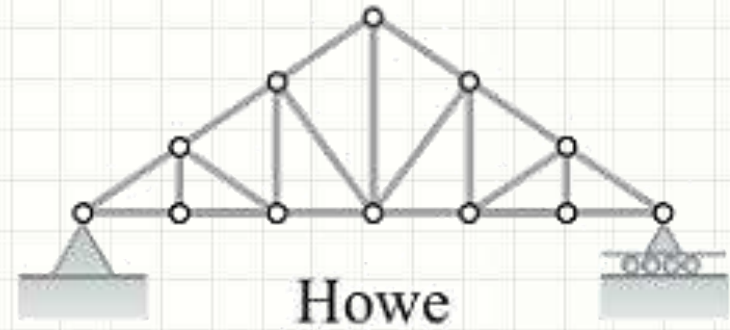
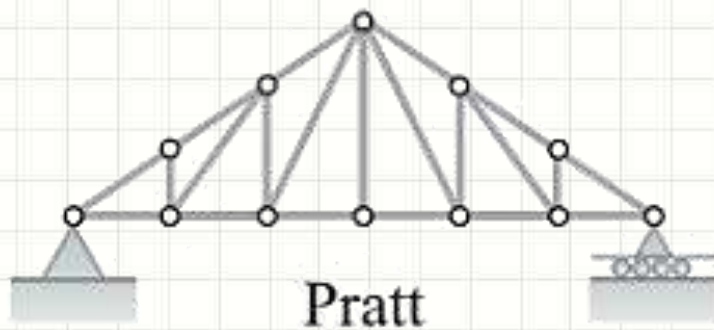
- Seguindo as regras elencadas
  - Barras tracionadas ou comprimidas
  - Nunca flexionadas!





# Tipos de Treliças Tradicionais

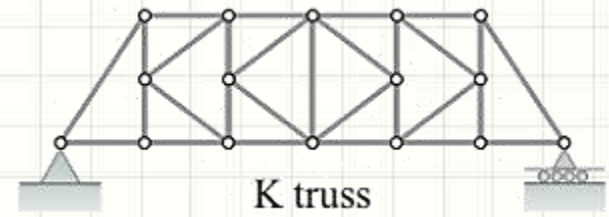
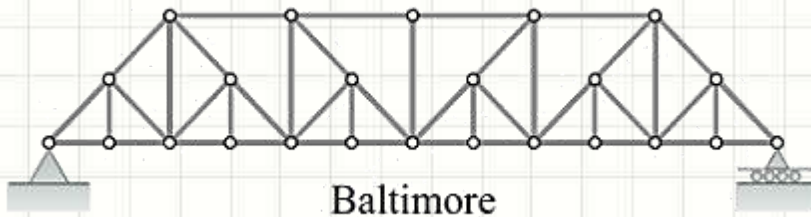
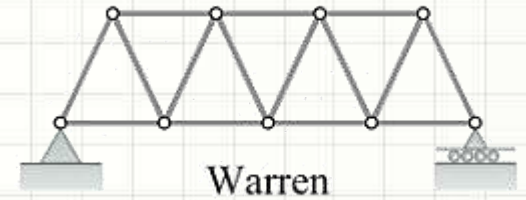
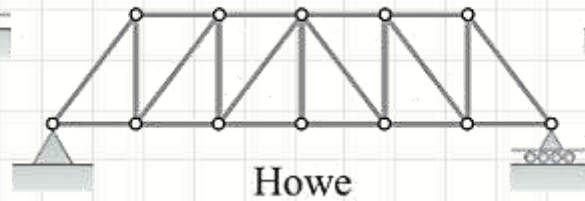
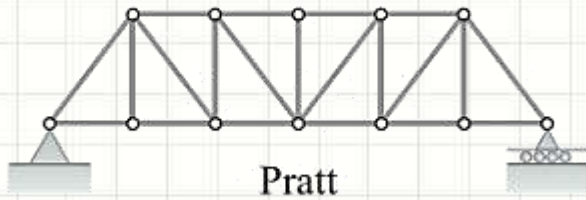
- Telhados





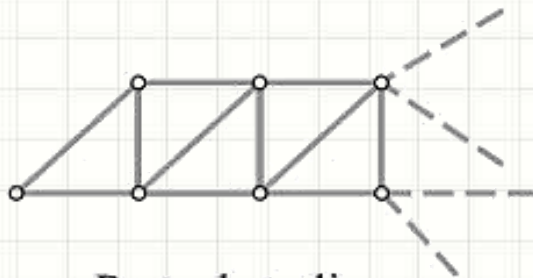
# Tipos de Treliças Tradicionais

- Pontes

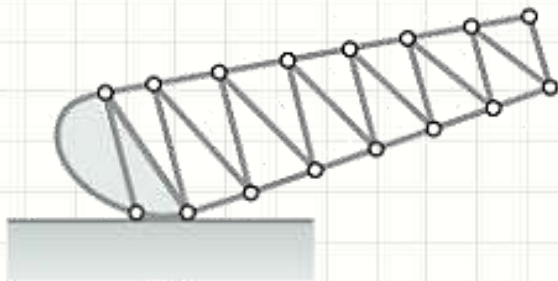


# Tipos de Treliças Tradicionais

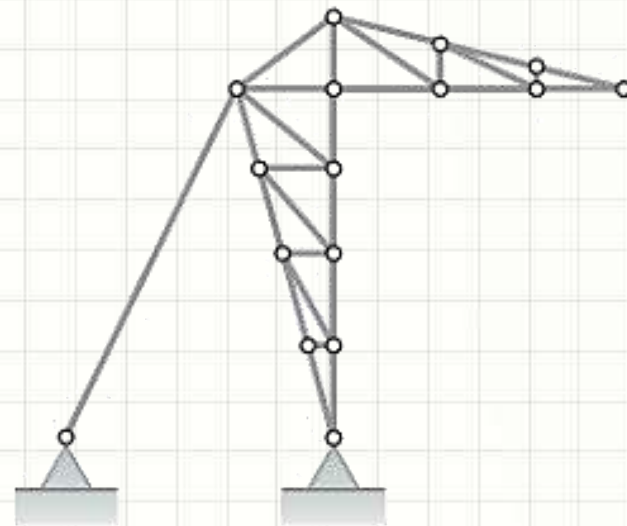
- Outros



Parte de treliça em balanço



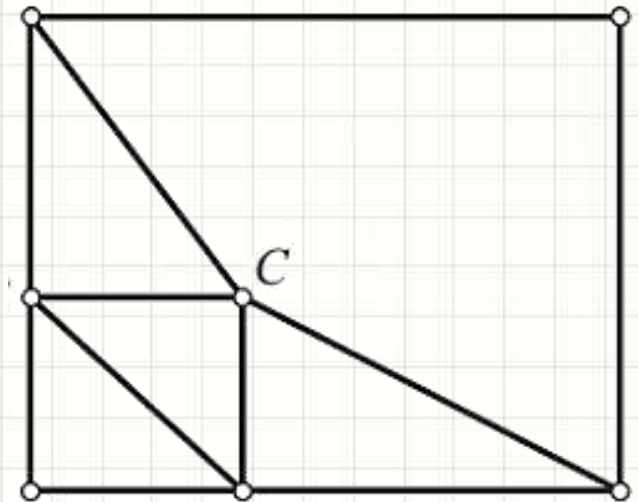
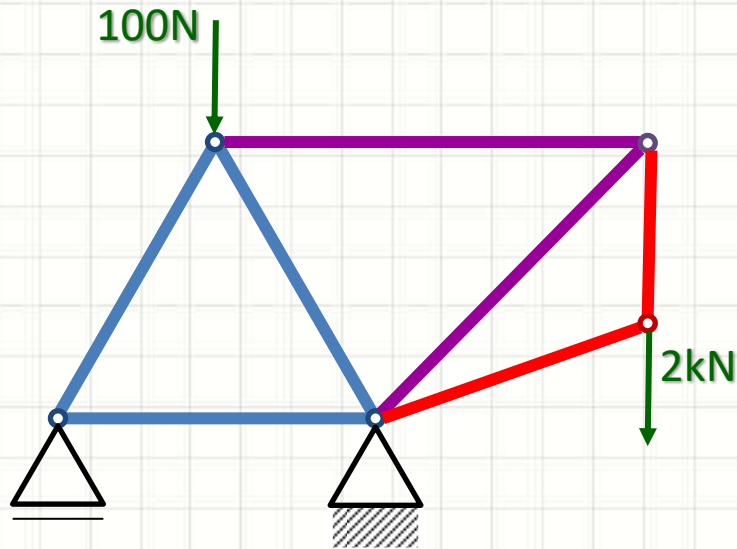
Basculante



Tipo estádio

# Treliças Simples

- Construídas a partir de um triângulo inicial
  - Cresce com a adição de barras em dupla
  - Apoiadas em dois dos nós e criando um novo

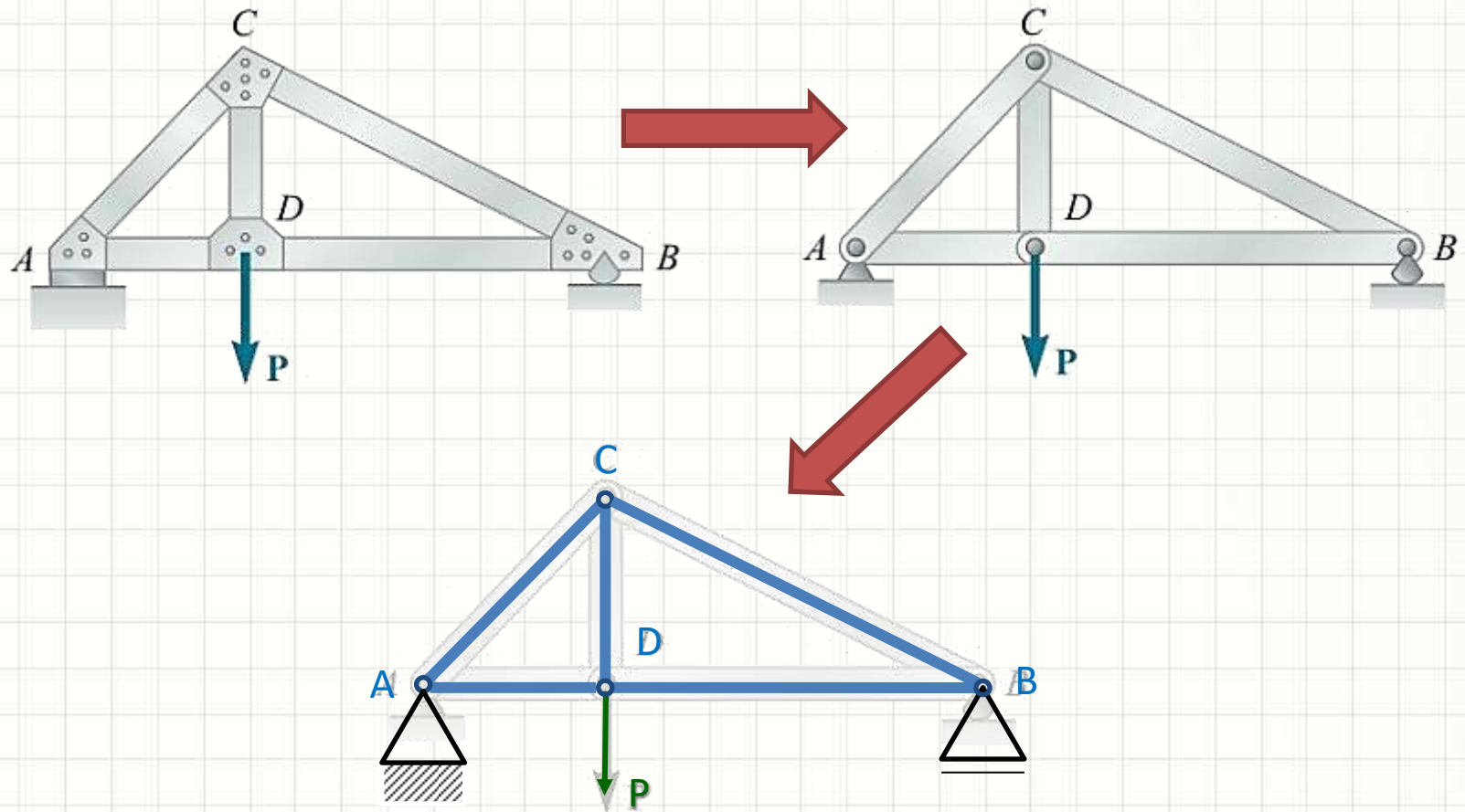




# **REPRESENTAÇÃO DOS COMPONENTES**

# Representação

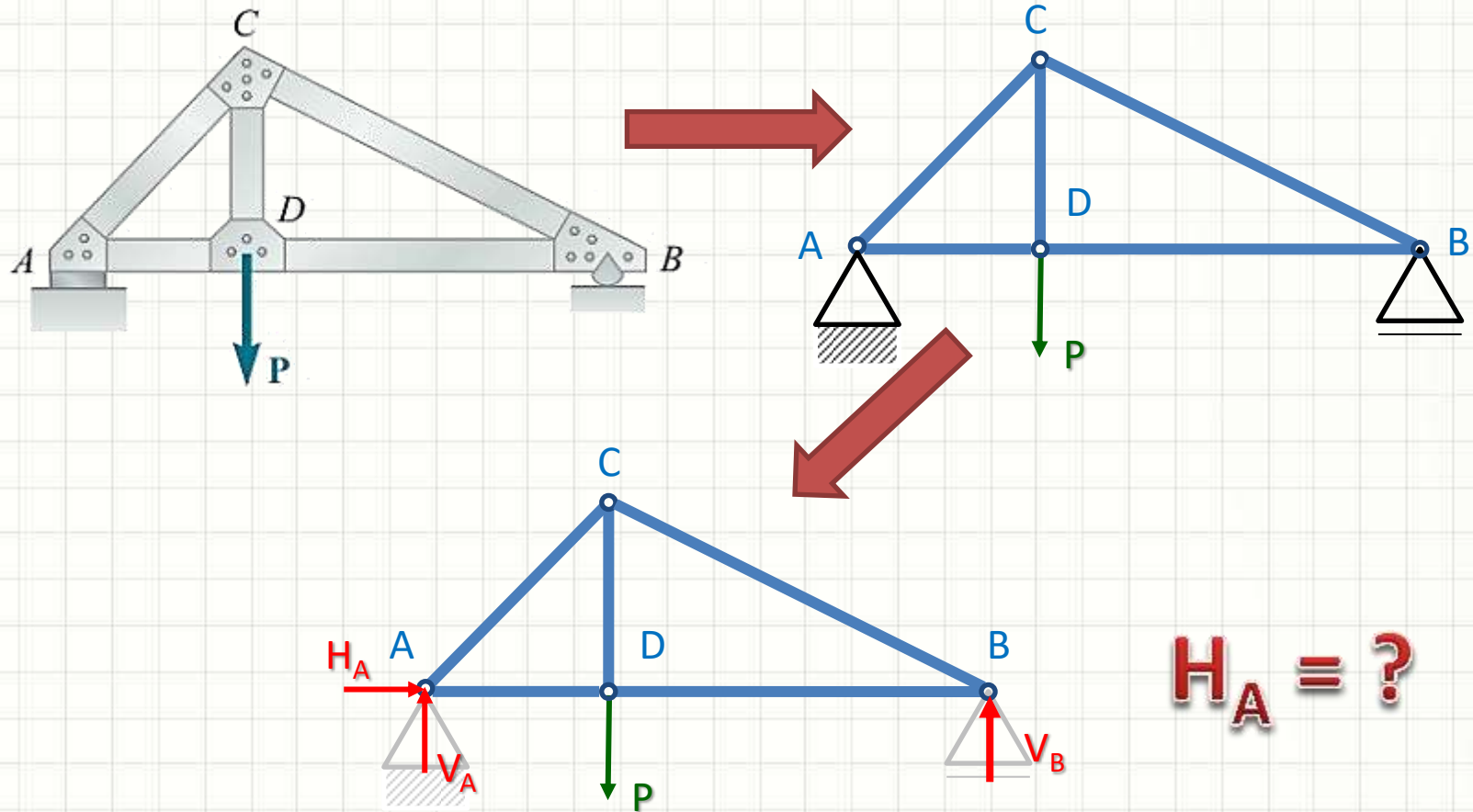
- Tomemos como exemplo a estrutura real



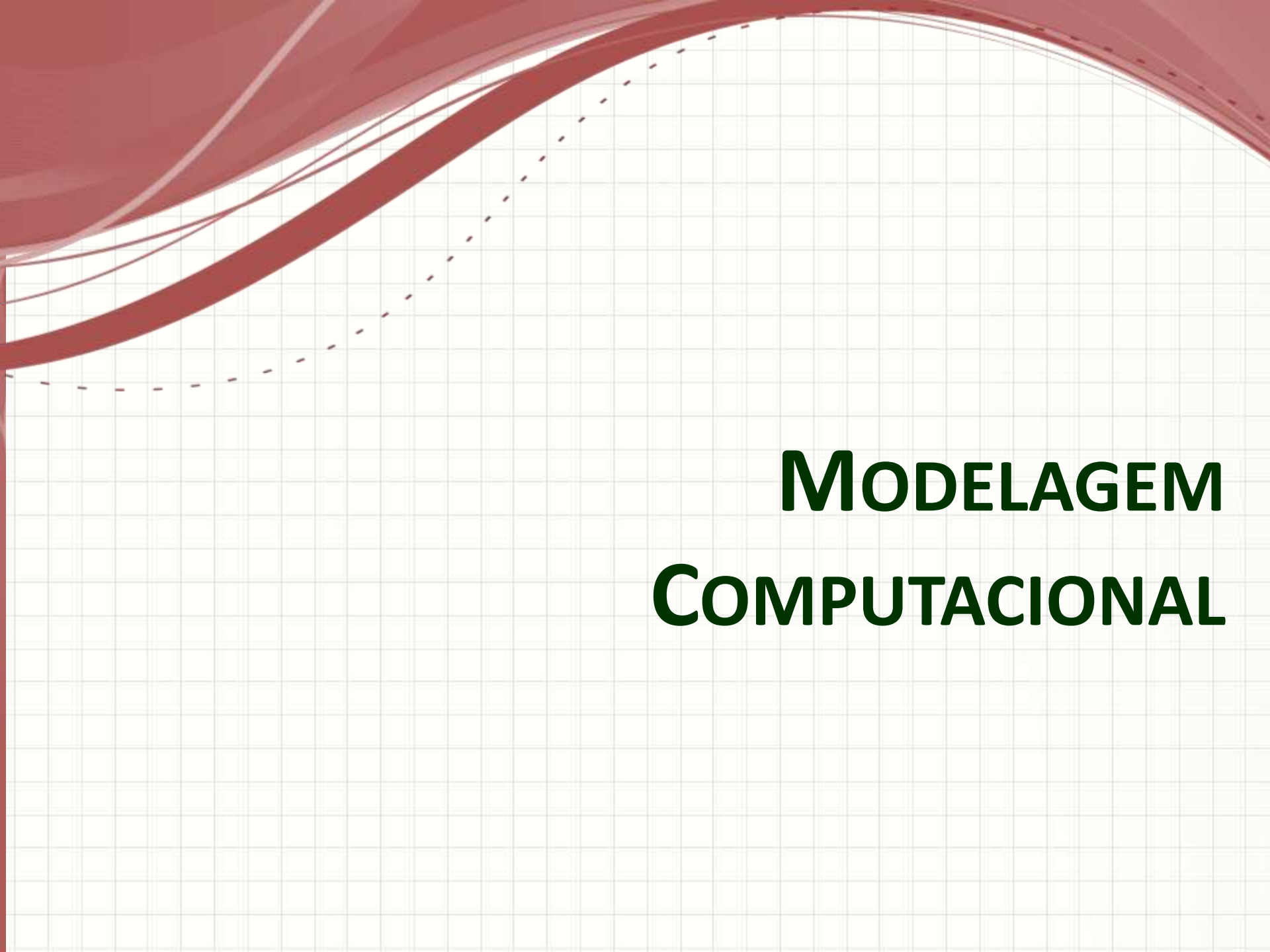


# Representação

- Tomemos como exemplo a estrutura real



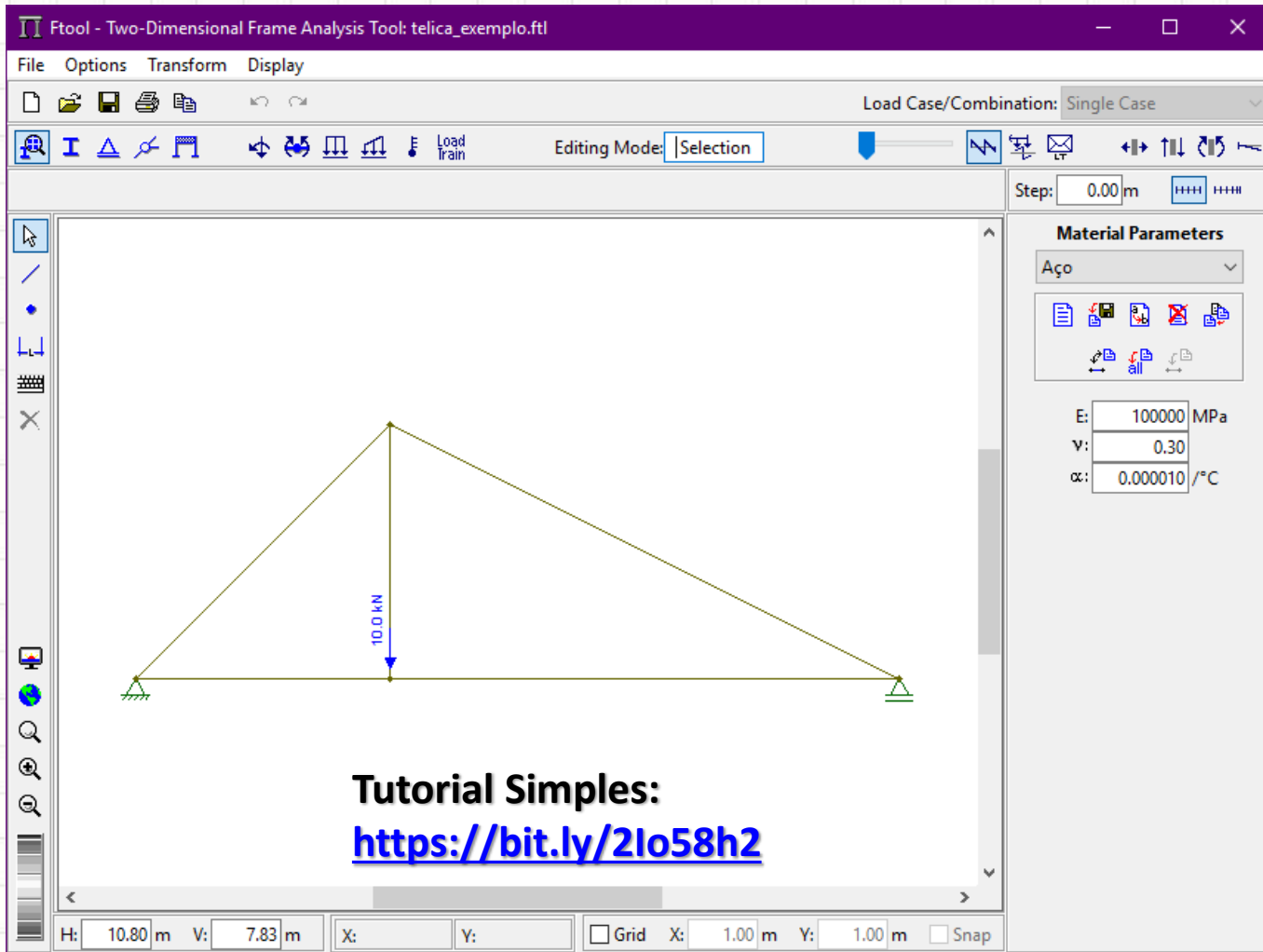
$H_A = ?$

The background features a light gray grid pattern. In the upper left corner, there are several overlapping, wavy red lines of varying thickness and opacity, creating a dynamic, abstract design. A dashed red line also curves across the upper portion of the grid.

# **MODELAGEM COMPUTACIONAL**

# Treliças: Modelo Computacional

- Programas: cálculo de estruturas de barras

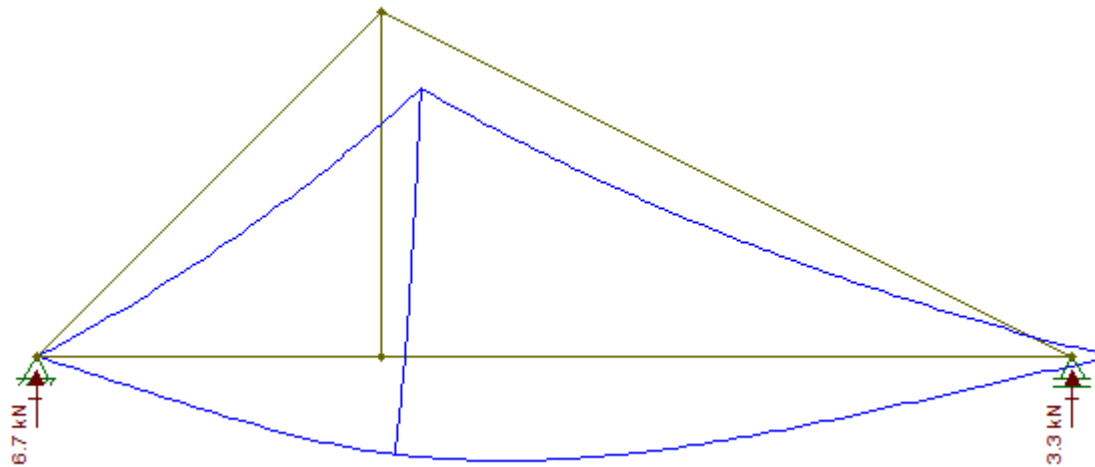
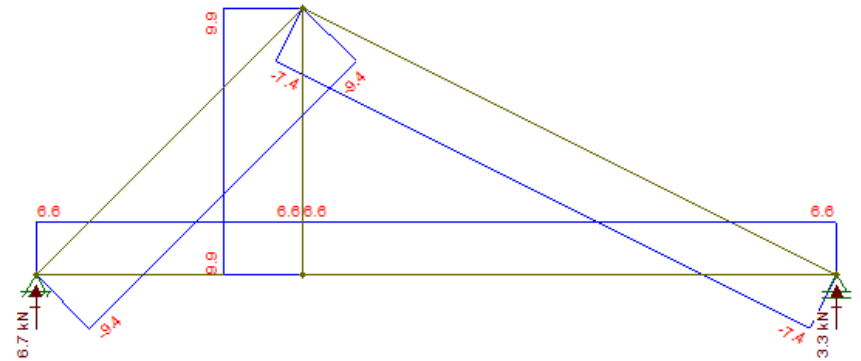
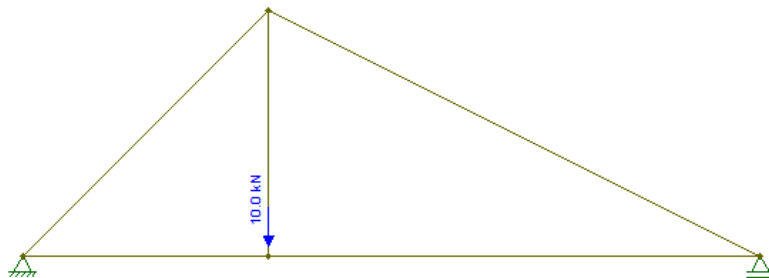


The screenshot displays the Ftool software interface for a two-dimensional frame analysis. The main window shows a truss structure with a horizontal base and two inclined members meeting at a peak. A vertical load of 10.0 kN is applied at the peak. The software interface includes a menu bar (File, Options, Transform, Display), a toolbar with various icons, and a status bar at the bottom showing dimensions (H: 10.80 m, V: 7.83 m) and grid settings. On the right side, the 'Material Parameters' panel is visible, showing the material 'Aço' with properties: E = 100000 MPa, ν = 0.30, and α = 0.000010 /°C. The text 'Tutorial Simples:' and the URL <https://bit.ly/2lo58h2> are overlaid on the bottom part of the window.

**Tutorial Simples:**  
<https://bit.ly/2lo58h2>

# Treliças: Modelo Computacional

- Programas: cálculo de estruturas de barras



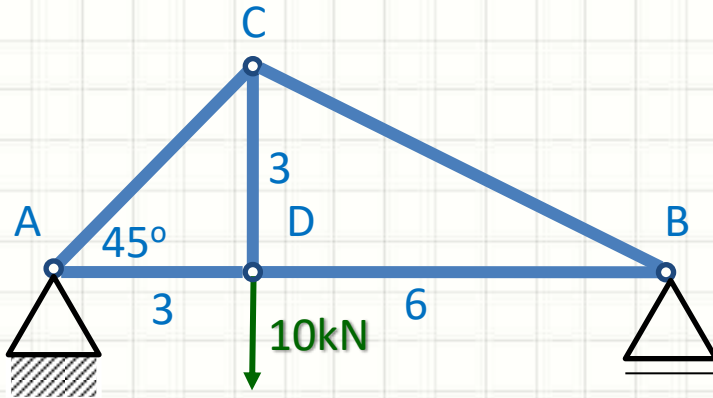


# **CALCULANDO AS REAÇÕES DE APOIO DA TRELIÇA**



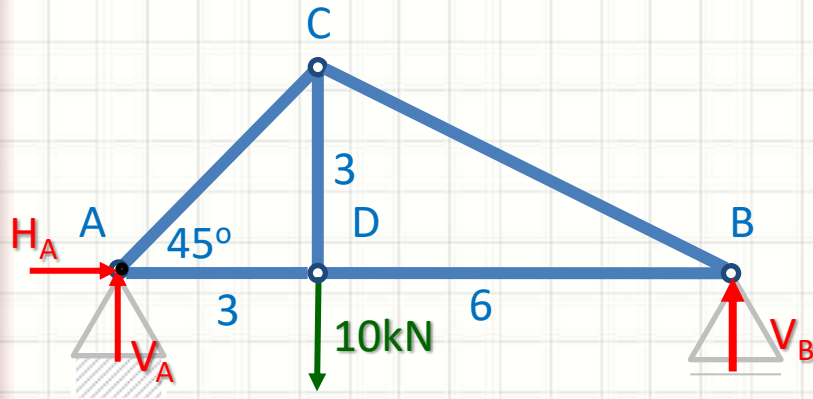
# Exemplo

- Calcule as reações de apoio



# Exemplo

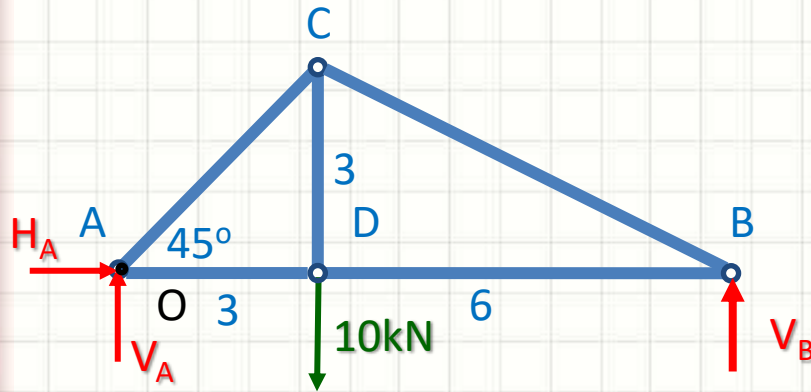
- Calcule as reações de apoio



1. Corpo livre

# Exemplo

- Calcule as reações de apoio



1. Corpo livre
2. Decompor esforços
3. Identificar as direções positivas
4. Determinar as reações

$$\sum F_x = 0 \Rightarrow +H_A + 0 = 0 \quad \Rightarrow H_A = 0N$$

$$\sum F_y = 0 \Rightarrow +V_A - 10000 + V_B = 0 \quad \Rightarrow V_A = 10000 - V_B$$

$$\sum M_O = 0 \Rightarrow +(10000 \cdot 3) - (V_B \cdot 9) = 0 \Rightarrow V_B = \frac{30000}{9} = 3,33kN$$

$$\therefore V_A = 6,67kN$$

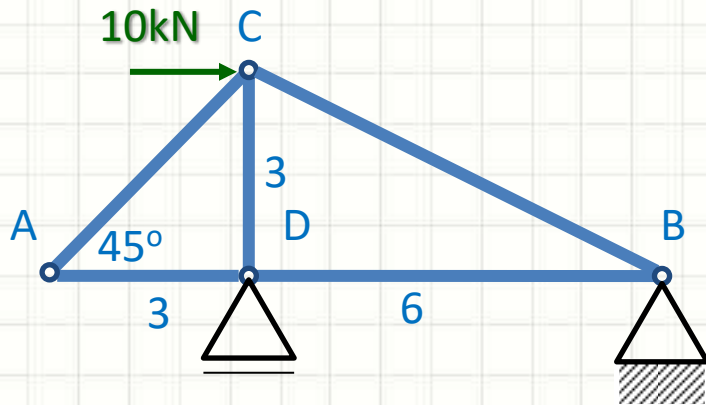
**Do ponto de vista dos apoios, a treliça se comporta como uma barra!**



# EXERCÍCIO

# Exercício

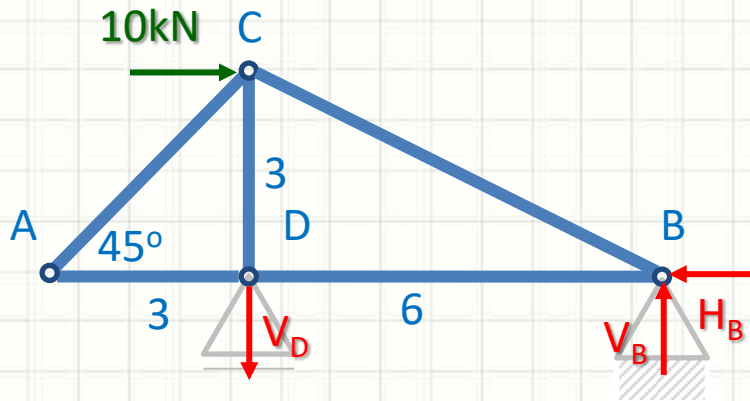
- Calcule as reações de apoio





# Exercício

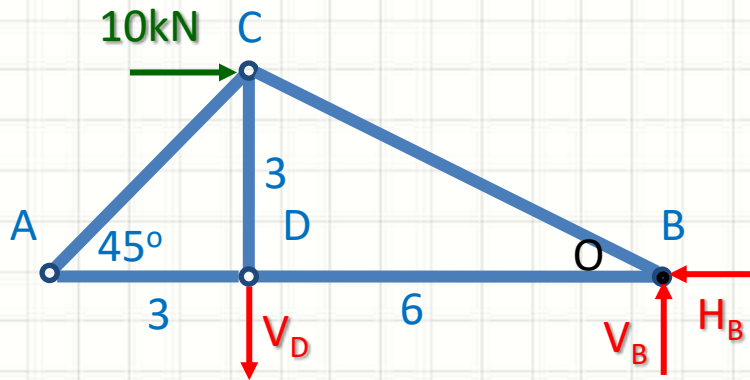
- Calcule as reações de apoio



1. Corpo livre

# Exercício

- Calcule as reações de apoio



1. Corpo livre
2. Decompor esforços
3. Identificar as direções positivas
4. Determinar as reações

$$\sum F_x = 0 \Rightarrow -H_B + 10000 = 0 \quad \Rightarrow H_B = 10kN$$

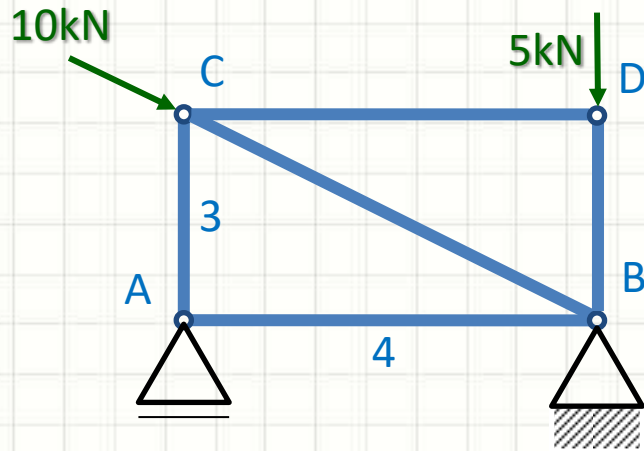
$$\sum F_y = 0 \Rightarrow -V_D + V_B = 0 \quad \Rightarrow V_B = V_D$$

$$\sum M_o = 0 \Rightarrow +(10000 \cdot 3) - (V_D \cdot 6) = 0 \Rightarrow V_D = \frac{30000}{6} = 5kN$$

$$\therefore V_B = 5kN$$

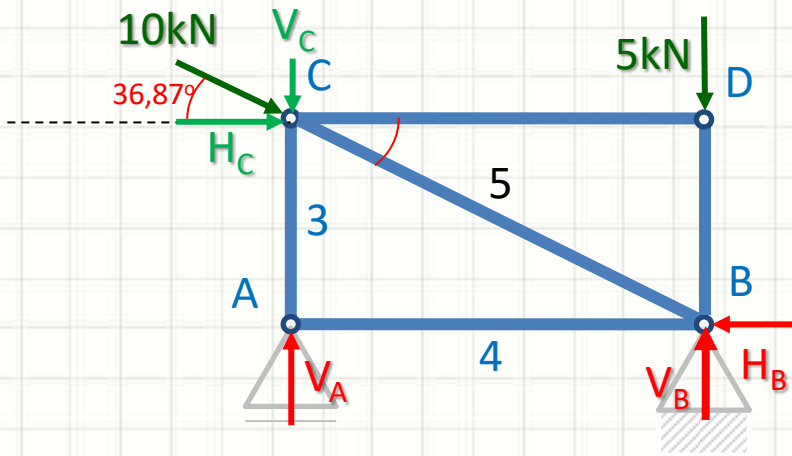
# Exercício

- Calcule as reações de apoio



# Exercício

- Calcule as reações de apoio



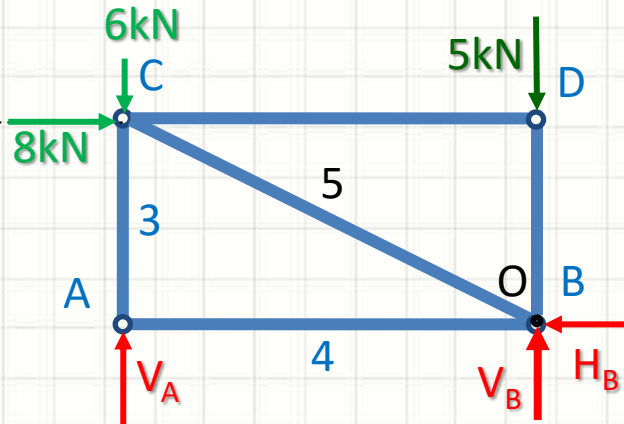
1. Corpo livre
2. Decompor esforços

$$H_C = 10000 \cdot \cos 36,87^\circ = 8kN$$

$$V_C = 10000 \cdot \sin 36,87^\circ = 6kN$$

# Exercício

- Calcule as reações de apoio



1. Corpo livre
2. Decompor esforços  
 $H_C = 10000 \cdot \cos 36,87^\circ = 8kN$   
 $V_C = 10000 \cdot \sin 36,87^\circ = 6kN$
3. Identificar as direções positivas
4. Determinar as reações

$$\sum F_x = 0 \Rightarrow +8000 - H_B = 0 \Rightarrow H_B = 8kN$$

$$\sum F_y = 0 \Rightarrow -6000 - 5000 + V_A + V_B = 0 \Rightarrow V_B = 11000 - V_A$$

$$\sum M_O = 0 \Rightarrow +(V_A \cdot 4) + (8000 \cdot 3) - (6000 \cdot 4) = 0 \Rightarrow V_A = 0kN$$

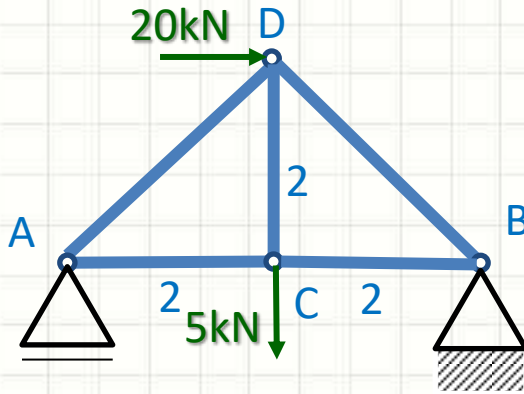
$$\therefore V_B = 11kN$$

**Será?**



# Exercício (para entrega!)

- Calcule as reações de apoio



$$V_A = 7,5kN$$

$$H_B = 20kN$$

$$V_B = 12,5kN$$



# CONCLUSÕES

# Resumo

- O que é uma treliça
- Características de uma treliça
- Tipos de treliça
- Cálculo de reações de treliça
- Modelagem computacional de treliça
- **TAREFA:** Exercícios Aula 4

**SAVA5!**

- 
- Estudo de treliças!
- 
- Esforços internos na treliça?
    - Como calcular?



**PERGUNTAS?**